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BUREAU OF EDUCATIONAL RESEARCH
COLLEGE OF EDUCATION

THE TEACHER'S
RESPONSIBILITY FOR DEVISING
LEARNING EXERCISES
IN ARITHMETIC

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PREFACE

The research reported in this monograph deals with a very practical problem. Every teacher of arithmetic continually faces the problem, "What learning exercises should I ask my pupils to do?" It is true that few teachers explicitly formulate this question but all must answer it. Incidentally it may be noted that the answer given is a very potent factor in determining the efficacy of the instruction.

Attempts to answer questions that ask what *should be* done may be designated as "complete research" to distinguish such investigations from fact-finding inquiries which may be called "auxiliary research." The work reported in this monograph represents an attempt to carry out a piece of "complete research." In this endeavor the results of a number of "auxiliary" or "fact-finding" studies have been utilized but reference to them has been subordinated to the consideration of the two basic problems. Even the report of the analysis of ten series of arithmetic texts, which represents more than 2500 hours of work, is made incidental to the solution of these problems.

A critical reader will probably be impressed by the incompleteness of the data needed for definite answers to the two basic questions. This condition is due in part to the complexity of these apparently simple problems but the available fact-finding studies relating to them furnish only fragments of the data necessary for detailed solutions. Many more auxiliary studies must be made before we can have what is commonly called a scientific answer to the question, "What is the responsibility of a teacher of arithmetic for devising and selecting learning exercises?"

It may even occur to the critical reader that an attempt to answer this question is not justified at the present time because the answer must be based upon fragmentary data and consequently judgment must be introduced at many places. In reply to this criticism, one may point out that every teacher is forced to give some answer to the question. Furthermore, if research workers become aware of the inadequacy of data for dealing with such practical questions, it is possible that they may be stimulated to group their fact-finding studies about certain fundamental problems. The justification of auxiliary studies is based upon the contributions they make to the solution of problems that ask "what should be."

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May 14, 1926.

THE TEACHER'S RESPONSIBILITY FOR DEVISING LEARNING EXERCISES IN ARITHMETIC

CHAPTER I

THE IMMEDIATE OBJECTIVES OF ARITHMETIC

The problem. The basic problems to be considered in this monograph are to determine (1) the nature and extent of the learning exercises¹ provided by texts in arithmetic and (2) the responsibility of the teacher for supplementing a text in this respect. In order to assist the reader in arriving at a clear understanding of these problems and to provide a basis for their consideration, two subordinate questions are treated: (3) what are the immediate objectives of arithmetic, and (4) what learning exercises are needed for the attainment of these objectives. The following pages of this chapter present an exposition of the objectives of arithmetic as a subject in the elementary school. Chapters II and III are devoted to a consideration of learning exercises and their relation to objectives. The explicit treatment of the two basic problems is given in Chapters IV and V.

A general statement of the objectives of arithmetic. The purpose of instruction in arithmetic is to engender in pupils the mental equipment needed for responding satisfactorily to certain types of quantitative situations which they will encounter in advanced school work and in life outside of school. This "mental equipment" is frequently called "ability in arithmetic." Sometimes the plural, "abilities," is used to indicate that the equipment is not a unitary thing but consists of a large number of elements, many of which are independent in the sense that a pupil may acquire certain ones but not others.

This general statement, like others which epitomize a group of concepts, will probably not have much meaning for the reader until the nature of the "mental equipment" and the situations for which it is to provide responses are described in some detail.

Types of arithmetical ability. Although psychologically all abilities have the common characteristic of a "bond" connecting a stimulus or situation and a response, and no sharp lines of demarcation can be

¹A learning exercise may be thought of as a request to do something. Examples and problems are prominent as learning exercises in arithmetic but as we shall show later (page 33) there are other types.

These problems assume that exercises are to be assigned by the teacher. See page 26.

specified, it is possible to identify three general types of ability; (1) specific habits, (2) knowledge, and (3) general patterns of conduct. The classification of a particular ability may not always be apparent, but the recognition of these rubrics will assist the reader in arriving at a clearer understanding of the immediate objectives to be attained by the teaching of arithmetic in the elementary school.

Nature of specific habits. If we examine the way in which pupils who have studied arithmetic respond to certain types of situations, we shall note certain distinguishing characteristics. For example, if a fifth-grade pupil is directed to write the numbers being dictated and "eighty-seven" is spoken, he writes the symbols "87" and does so without thinking, that is, automatically and mechanically. When a number symbol such as "4" is brought to his attention, a meaning² immediately comes into his mind. In other words the response, "meaning of the symbol 4," is connected with visual apprehension of the symbol so that, when the visual apprehension occurs, the meaning comes into consciousness and does so without the pupil making any effort to recall. If a sixth-grade pupil is asked "what is the product of 7×6 ?", 42 comes into his mind as a response. When he is asked "how many feet in a yard?", he answers "three." When such words as add. product, divide, multiply, equal, and the like are brought to his attention, either orally or in printed form, a meaning immediately comes into his mind.

For situations of the types illustrated in the preceding paragraph, one who has been "educated in arithmetic" possesses a ready-made response and is able to make it fluently, that is, quickly and with a minimum of conscious effort. Such mental equipment is usually designated as motor skills, fixed associations and memorized facts, or more briefly as *specific habits*. The word "specific" is used to indicate that each response is connected with only one situation and that any given situation requires a certain response.

The scope of specific habits in the field of arithmetic. The "tables," addition, subtraction, and so forth represent a number of facts that are to be memorized but the total number of specific habits in the field of arithmetic is much larger than is commonly realized. Investigation has shown that $6 + 7$ and $7 + 6$ form the basis of two specific habits instead of a single one. Additional specific habits are required for $17 + 6$ and $16 + 7$, $27 + 6$ and $26 + 7$, $36 + 6$ and $36 + 7$, and so

²There are three general types of meanings for number symbols: a name for a group of objects, a position in the number series and its ratio to other numbers. The meaning which a person associates with a particular number symbol may be a combination of these elemental meanings.

forth. One investigator,³ after careful inquiry, has concluded that there are 412 addition combinations which a child "is *almost sure* to need after he leaves school." This means that, if a child fails to learn any one of these 412 combinations, there will be a corresponding "gap" in his ability to add integers.

The specific habits which function in arithmetical calculations have been mentioned first, but they do not constitute all of this type of mental equipment. Pupils are expected to learn the meaning of number symbols, both integers and fractions. By implication this includes understanding the decimal system of notation and the ability to read and write numbers in Arabic notation. In addition to the symbols used in expressing numbers, the pupil is expected to learn several such as \$, ×, =, and a large number of technical and semi-technical terms including their abbreviations. The topic of denominate numbers furnishes a large group of such terms, but there are many others such as sum, product, remainder, percent, interest, profit, loss, balance, overdraft, discount, average, bill, rectangle, triangle, buy, sell, at the rate of, and per yard. Quantitative relationships such as the number of feet in a yard, number of quarts in a gallon, the fractional equivalent of 12½%, and the like furnish the basis for another group of specific habits.

The nature of abilities designated as knowledge. Specific habits provide controls of conduct for responding to familiar situations. When such situations are encountered, the pupil "remembers" the responses he found to be satisfactory on previous occasions. When "new" situations⁴ are encountered, a pupil's specific habits are inadequate as controls of conduct. He must manufacture a response using the ideas, facts, concepts, and principles that the "new" situation suggests to him. This mental equipment is called knowledge⁵ and the process of using it is designated as reasoning or reflective thinking.

It is not possible to specify a sharp line of demarcation between specific habits and knowledge. The connection between a meaning, concept, or principle, and a given situation may be fixed through repetition so that the control of conduct is changed from knowledge to a specific habit. The degree of the strength of the bond connecting a response with a situation is, however, not the most significant basis of

³OSBURN, W. J. *Corrective Arithmetic*. Boston: Houghton Mifflin Company, 1924, p. 21.

⁴A "new" situation may, and frequently does, involve familiar elements but the total combination is one to which the pupil has not responded before or one for which he has forgotten the response.

⁵This definition of knowledge indicates a more restricted meaning than is commonly associated with the term.

distinction between specific habits and knowledge. The latter rubric of controls of conduct is characterized by many associations which result in "richness of meaning," and by organization which ties together the items of knowledge so that the recall of one item will tend to bring related items in one's mind. For example, assume that the following problem is "new" to a sixth-grade pupil: "The expenses of running a grocery store amount to 20% of the receipts. How much must a grocer charge for a barrel of flour which costs him \$15.00 in order to make a net profit equal to 10% of the selling price?"

Although this problem is "new" to the pupil, it involves familiar elements (words and phrases) to which he responds by recalling ideas, concepts, and principles. These in turn may suggest other items of knowledge. Out of the total ideas, concepts, and principles that are active in his consciousness, the pupil formulates a tentative response to the problem. This is tested and if found unsatisfactory another formulation is made and tested. The solving of the problem is characterized by deliberation rather than fluency. This description of knowledge does not constitute a detailed definition but it is sufficient for our present purpose, which is to point out that knowledge is included in the outcomes of arithmetical instruction.

General patterns of conduct as mental equipment. Specific habits and knowledge do not suffice as categories for classifying all abilities resulting from the study of arithmetic. Neatness, accuracy, systematic attack, persistence, and the like designate controls of conduct which are sometimes called habits or general habits. However, they differ in significant respects from "specific habits" described earlier. Accuracy or "habit of accuracy" does not designate a response to a particular situation. It is rather a general mode or pattern of response to many situations. Neatness in calculating is not a response to a particular example such as "divide 846.84 by 396" but rather a general pattern of response in performing all calculations. In order to emphasize this distinction, the name "general pattern of conduct" is given to such controls of conduct as neatness, accuracy, and so forth.

Another aspect of the aim of arithmetic. The preceding discussion has pointed out three types of mental equipment which teachers of arithmetic are expected to engender; first, specific habits that function in making calculations and in responding to certain other types of situations; second, knowledge out of which pupils will be able to construct responses to "new" problems which they will encounter in other school activities and in life outside of school; and third, general patterns of conduct. This analysis of "ability in arithmetic" has added meaning to

the general statement of aim with which we began, but the types of situations for which pupils are to be equipped by the study of arithmetic have been indicated only in very general terms. A complete understanding of the immediate objectives of arithmetic requires specifications in regard to what specific habits, what items of knowledge and what general patterns of conduct are to be engendered by the instruction in this school subject, and the quality of each ability.⁶

Determination of the particular arithmetical abilities to be engendered. A method of determining the specific habits, items of knowledge and general patterns of conduct that should be engendered by instruction in arithmetic is suggested by the statement of the general purpose given on page 7. If this instruction is to engender the [mental equipment needed for responding satisfactorily to certain types of quantitative situations] which will be encountered in advanced school work and in adult life, it appears logical to analyze advanced school work and adult life for the purpose of determining the quantitative situations involved.⁷ With this information at hand, additional analyses should reveal the nature and extent of the arithmetical equipment needed for making satisfactory responses.

A number of analyses of adult activities have been made. One of the most elaborate is by Wilson⁸ who collected from adults 14,583 arithmetical problems which they had encountered in their activities. From his analysis of these problems Wilson reached certain conclusions relative to the arithmetical equipment that adults need. For example, the demand for the equipment engendered by the study of the following topics is so slight that he recommends their elimination:

1. Greatest common divisor and least common multiple beyond the power of inspection.
2. Long, confusing problems in common fractions.
3. Complex and compound fractions.
4. Reductions in denominate numbers.
5. Table of folding paper, surveyors table, tables of foreign money.
6. Compound numbers, neither addition, subtraction, multiplication nor division.
7. Longitude and time.

⁶The quality of arithmetical abilities is considered on page 15.

⁷This method of determining educational objectives is called "job analysis."

⁸Wilson, G. M. "The social and business usage of arithmetic." Teachers College Contributions to Education, No. 100. New York: Teachers College, Columbia University, 1919. 62 p.

8. Cases 2 and 3 in percentage.
9. Compound interest.
10. Annual interest.
11. Exchange, neither domestic nor foreign.
12. True discount.
13. Partnership with time.
14. Ratio, beyond the ability of fractions to satisfy.
15. Most of mensuration,—the trapezoid, trapezium, polygons, frustum, sphere.
16. Cube root.
17. The metric system.

Wilson's study also yielded information relative to the character of the calculations made by adults in solving the problems they encounter. Slightly more than half of the additions involved either one or two place addends and less than two percent involved addends of more than four places. An analysis of a portion of the problems showed that nearly a third (31.2 percent) of the additions involved only two addends and that less than seven percent involved more than six addends. Subtractions, multiplications and divisions were also shown to be relatively simple. There were only 1,974 occurrences of common fractions in the 14,583 problems and ten different fractions accounted for in 95.5 percent of the cases.⁹

In summarizing his conclusions Wilson states: "If to the four fundamentals and fractions one were to add accounts, simple denominate numbers, and percentage, little would be left for all the other processes,—so little in fact that it seems unfair to give attention to them as drill processes in the elementary schools. Some of them should receive no attention. Others should receive attention only for informational purposes or when found necessary in the development of motivated situations."

Limitations of the job-analysis procedure. Several other investigators employing similar methods,¹⁰ have contributed information in regard to the arithmetical equipment which adults use in their activities and it may appear that such analyses when sufficiently extended will

These fractions in order of frequency are $\frac{1}{2}$, $\frac{3}{4}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{3}{8}$, $\frac{2}{5}$, $\frac{4}{5}$, $\frac{1}{8}$ and $\frac{1}{5}$.

⁹ADAMS, H. W. "The Mathematics Encountered in General Reading of Newspapers and Periodicals." Unpublished Master's thesis, Department of Education, University of Chicago (August, 1924). Reviewed by BOBBITT, FRANKLIN K., in *Elementary School Journal*, 25:133-43, October, 1924.

CAMERER, ALICE. "What should be the minimal information about banking?" Third Report of the Committee on Economy of Time in Education. Seventeenth Year-

yield a complete and dependable inventory of the arithmetical equipment which our schools should endeavor to engender. However, the job-analysis method of determining educational objectives has certain limitations which should be noted. In the first place the functioning of arithmetical equipment is not confined to the solving of problems or the making of calculations. As one comprehends numbers, names of denominate quantities, and other items of arithmetical terminology either in listening to a speaker or in reading, he is using elements of his arithmetical equipment. Furthermore, not infrequently one has occasion to estimate magnitudes such as the height of a tree, the number of tons in a pile of coal, and so forth, and to answer thought questions involving quantities but not requiring calculations. In both estimating magnitudes and answering quantitative thought questions, one uses arithmetical equipment along with other controls of conduct.

A second point is that the present activities of adults do not necessarily include all of the uses of arithmetical equipment that should be made. For example, authorities urge that farmers keep a detailed account of their financial activities; that individuals keep personal accounts; and that heads of families plan a budget at the beginning of the year and conform to it as closely as possible. However, these activities are not engaged in by all persons to whom they apply. In fact it is doubtful if they are engaged in at all generally.

A third point to be noted is that some activities requiring arithmetical equipment are engaged in by practically all adults but other activities are highly specialized. For example, everyone has occasion to count money and to check the making of change by clerks and storekeepers. Most adults have a bank account and should keep the stub of their check book. On the other hand, relatively few adults engage

book of the National Society for the Study of Education, Part I. Bloomington, Illinois: Public School Publishing Company, 1918, p. 18-26.

MACLEAR, MARTHA. "Mathematics in current literature," *Pedagogical Seminary*, 30:48-50, March, 1923.

MITCHELL, H. EDWIN. "Some social demands on the course of study in arithmetic." Third Report of the Committee on Economy of Time in Education. Seventeenth Yearbook of the National Society for the Study of Education, Part I. Bloomington, Illinois: Public School Publishing Company, 1918, p. 7-17.

NOON, PHILO G. "The child's use of numbers," *Journal of Educational Psychology*, 10:462-67, November, 1919.

SMITH, NILA B. "An investigation of the uses of arithmetic in the out-of-school life of first-grade children," *Elementary School Journal*, 24:621-26, April, 1924.

WISE, CARL T. "A survey of arithmetical problems arising in various occupations," *Elementary School Journal*, 20:118-36, October, 1919.

WOODY, CLIFFORD. "Types of arithmetic needed in certain types of salesmanship," *Elementary School Journal*, 22:505-20, March, 1922.

TABLE I. OCCUPATIONAL DISTRIBUTION OF PERSONS TEN YEARS OF AGE AND OVER, 1920 FEDERAL CENSUS, AND DISTRIBUTION OF PROBLEMS IN FOUR SERIES OF ARITHMETICS WITH RESPECT TO THE ACTIVITY IN WHICH THEY AROSE

Occupational Division	Percent of population	Percent of Problems in Four Texts
Agriculture, forestry and animal husbandry.....	26.3	11.2
Extraction of minerals.....	2.6	0.1
Manufacturing and mechanical industries.....	30.8	10.3
Transportation.....	7.4	5.3
Trade.....	10.2	43.7
Public service (not elsewhere classified).....	1.9	3.4
Professional service.....	5.2	1.3
Domestic and personal service.....	8.2	0.1
Clerical occupations.....	7.5	0.1
Activities of the home.....	6.0
Personal activities.....	17.0
Activities of school children.....	1.6
Total.....		100.1

in certain vocational activities that provide many arithmetical problems. In the 1920 Federal Census, 572 occupations and occupational groups were used in classifying the persons employed in gainful occupations. The distribution of persons of ten years and over among the nine occupational divisions is shown in Table I. The largest percent (30.8) is for "manufacturing and mechanical industries" but a large proportion of those engaged in this division of occupations are listed as laborers or semi-skilled employees. An analysis¹¹ of the problems of four series of arithmetic texts with respect to their source gave the distribution shown in the last column of Table I. Obviously, "trade" is the principal source of problems although it is engaged in by only about one person in ten. In a more detailed table that is not reproduced here, it is shown that in 1910 approximately 55 percent of our population of ten years of age and over were engaged in occupations to which no arithmetical problems found in the texts examined could be assigned.

The facts presented in Table I suggest that the analysis of occupational activities for the purpose of identifying the arithmetical prob-

¹¹MONROE, WALTER S. "A preliminary report of an investigation of the economy of time in arithmetic." Second Report of the Committee on Minimal Essentials in Elementary School Subjects. Sixteenth Yearbook of the National Society for the Study of Education, Part I. Bloomington, Illinois: Public School Publishing Company, 1917, p. 111-27.

lems that occur has a very limited value. The need for the arithmetical equipment necessary to meet situations arising in particular occupations may be greater than these facts indicate. When considering educational objectives one should recognize that the general public may be considered as sustaining a "consumer's" relation to a number of occupations. For example, only one tenth of our adult population is engaged in trade occupations but practically everyone engages in buying and therefore has occasion to check sale's slips, count change, and so forth. It is not possible for us to know in advance just which children will become clerks, which ones farmers, which ones stenographers, which ones machine operators, and so forth. Furthermore, persons engaged in one occupational activity should know something about the work of others. Not only is there considerable transfer of workers from one occupation to another, but social solidarity requires mutual understanding and respect, and the more the workers in one occupation know of other occupations the greater will be their capacity for understanding and respecting their fellowmen.

Conclusion in regard to what arithmetical abilities should be engendered. The considerations just noted suggest that job-analysis studies are not likely to yield precise and complete determinations of the particular abilities to be engendered by instruction in arithmetic. Studies already made indicate the elimination of certain abilities formerly included among the objectives of arithmetic. Other studies have indicated the inclusion of new abilities or increased emphasis on certain abilities already included. Future studies will probably contribute to still further refinements of arithmetical objectives but the limitations noted should not be overlooked. For the present we are able to compile an inventory only in general terms of the arithmetical abilities to be engendered.

The quality of arithmetical equipment. Another aspect of the objectives of arithmetic relates to the quality of the controls of conduct to be engendered. In the case of specific habits the quality of an ability is usually described in terms of rate and accuracy. For example, if one describes a pupil's ability to do addition examples of a given type, he specifies the rate at which the addition is done and the degree of accuracy of the sums. The idea of both rate and accuracy is frequently combined in the single term "fluency."¹²

¹²For a partial specification of fluency see:

HERRIOTT, M. E. "How to make a course of study in arithmetic." University of Illinois Bulletin, Vol. 23, No. 6, Bureau of Educational Research Circular No. 37. Urbana: University of Illinois, 1925, p. 10, 11, 29, and 37.

Another phase of the quality relates to the "degree of permanency." If we assume that it is desirable for a student to acquire a certain ability, how long should he be expected to retain this ability? A pupil may learn a denominate number relation or the meaning of a technical term well enough so that he will remember it for a week. Additional learning will result in his retention of the control of conduct until the end of the school year and if the learning is continued sufficiently the control of conduct will tend to become a relatively permanent acquisition.

A description of the specific habit and knowledge objectives of arithmetic.¹³ The preceding discussion has indicated the difficulties encountered in preparing a complete and detailed inventory of the specific habits and items of knowledge to be engendered by instruction in arithmetic. As yet our information concerning the demands for arithmetical equipment is so limited that such an inventory cannot be formulated. However, it is possible to describe in some detail the types of situations which children and adults encounter and to indicate the nature of the response to be made. Although such a description will be subject to the limitation that the range of situations within each type is not determined except in a general way, the enumeration of types should lead the reader to enrich his concept of the objectives of arithmetic. No attempt is made to specify the quality (fluency or permanency) of the abilities necessary for satisfactory responses, nor to distinguish between specific habits and knowledge.

I. Number symbols. These include Arabic symbols 0, 1, 2, 3 9; numbers expressed in the decimal notation, 10, 11, 12 100, 101, 102, 1000, 1001,; common fractions and mixed numbers; decimals .5, .05, 12.5, $.66\frac{2}{3}$, .6666; Roman numerals; and verbal expressions of numbers (both oral and written), zero, one, two, one hundred, one-half, first, second, and so forth.

Responses to be connected with number symbols. The outstanding type of response to be made to number symbols is designated in a general way by the term "meaning" but the nature of this response need not be always the same for the same type of symbol. For "small" integers the meaning should sometimes include an image of a definite group of objects or a rather precise idea of the relations of the integer to other integers. In the case of "large" numbers a less definite meaning is expected. Usually a picture or idea of the position of a number in the number system provides an adequate control of conduct. Children should learn "definite" meanings for the more commonly used

¹³General patterns of conduct are considered on page 23.

fractions, both common and decimal, and “less definite” ones for the fractions that are encountered infrequently. Meaning responses should be connected with the Arabic number symbols, verbal expressions of numbers, oral, printed or written, and with the more commonly used Roman numerals.

The conventional oral responses should be connected with the visual apprehension of the various printed or written number symbols and the conventional written responses with the auditory apprehension of spoken number symbols. These two types of equipment are required for reading numbers and for writing them from dictation. Another type of equipment is needed for copying numbers.

II. Other arithmetical symbols and technical terms. In addition to number symbols, certain conventional signs such as $+$, $-$, \times , \div , $=$, and $\%$ are employed in arithmetic. Closely related to these are the conventional arrangements of the number symbols indicating calculations. For example, numbers written in a column with the right hand margin even indicate addition. The following arrangement indicates the division of 576 by 36.

$$\begin{array}{r} 16 \\ 36 \overline{)576} \\ \underline{36} \\ 216 \\ \underline{216} \\ 0 \end{array}$$

The technical terms include (1) those relating to calculation such as add, multiply, sum, remainder, quotient, partial product, “times” as in 6 times 7, “of” as in $\frac{3}{4}$ of 8, total, and average; (2) names of denominate numbers and their abbreviations such as quart, foot, pound, barrel, dollar, and so forth; (3) terms relating to quantitative aspects of certain adult activities such as account, interest, balance, amount, change, profit, premium, rectangle, circle, and area; (4) words and phrases such as how many, how much, and, each, remains, bought, sells, lost, earns, what is, find (the sum, product, etc.), and the like. The terms in the fourth group are not peculiar to arithmetic but in problems they frequently have a technical meaning. Sometimes they are designated as semi-technical terms.

Responses to be connected with “other arithmetical symbols and technical terms.” The outstanding type of response to be made to this class of situations and stimuli may also be designated as “meaning” but often the meaning of symbols and terms relating to calculation is

evidenced by a motor response as in "Find the product of 2894 and 672." The child is expected to respond to this situation by writing $\begin{array}{r} 2894 \\ 672 \end{array}$ and not $672 \overline{)2894}$ or some other arrangement of the numbers.

III. Two or more numbers quantitatively related with one missing. The simplest situations under this head are commonly designated as the "tables" or "basic" combinations such as $9 + 3 =$, $7 + 0 =$, 5 (add) 8 (add) 9 (subtract) 6 0 $7 - 3 =$, $5 - 0 =$, 4 $9 \times 5 =$, 8 (multiply)

4 $12 \div 4 =$, $9 \overline{)36}$. Until recently it has been assumed that the 100 addition combinations¹⁴ represented all of the addition situations involving only integers for which responses should be memorized. It now appears that the addition situation $6 + 7$ cannot be considered as essentially the same as $16 + 7$, $26 + 7$, and so forth, and therefore 300 or more "higher decade" combinations must be added to the 100 "basic" combinations. A limited number of "higher decade" subtraction combinations occur in short division. There are no additional combinations in multiplication. A feature of the "higher decade" combinations in addition and subtraction is that one of the numbers may be an "inner stimulus," an idea, and not something seen or heard. For example, in adding the column of figures shown on the right, one sees the 7 and 3 but as he adds up the column he does not see the partial sums 10, 15, 24, and 31 to which the numbers 5, 9, 7 8 and 8 respectively are to be added. In division it appears likely 7 that responses should be "learned" for all situations having as 9 divisors 1, 2, 3, 4, 5, 6, 7, 8, or 9 and dividends ranging from 0 5 to those that are 10 times the divisor.¹⁵ This means that 3 $17 \div 2 =$, $13 \div 4 =$, and $52 \div 9 =$, represent combina- 7 tions as well as $16 \div 2 =$, $12 \div 4 =$ and $54 \div 9 =$. When considered in this way, division affords 360 additional combinations and each one can be expressed in two ways such as $17 \div 2 =$, and $2 \overline{)17}$.¹⁶

In addition to the three-number relationships described in the preceding paragraphs, there are a number of situations involving only two

¹⁴Investigation has revealed that $6 + 3 =$ and $3 + 6 =$ cannot be considered identical situations. Similar statements can be made with reference to subtraction, multiplication, and division. Hence there are 100 "basic" combinations in addition, subtraction and multiplication, and 90 in division.

¹⁵OSBURN, W. J. *Corrective Arithmetic*. Boston: Houghton Mifflin Company, 1924, p. 20.

¹⁶The combinations described in this paragraph may be called "secondary" to distinguish them from the "basic" ones commonly referred to as "the tables."

numbers; equivalent fractions, such as $\frac{1}{2} = \frac{2}{4}$, $\frac{6}{8} = \frac{3}{4}$, $.5 = \frac{1}{2}$, $\frac{1}{8} = .12\frac{1}{2}$ and denominate number relations such as 3 ft. = 1 yd., 1 bu. = 2150.42 cu. in.

Responses to be connected with quantitative relationships with one number missing. The outstanding response to be made to quantitative situations of the types described in the preceding paragraphs is the supplying of the missing number. Sometimes this response is to be expressed in written form; on other occasions it is to be partially written (e. g., units written and tens carried); and in column addition and multiplication the response may not be expressed but functions as an element in the next situation.

IV. Examples. The term "examples" is used to designate explicit requests to add, subtract, multiply, divide or extract a root, the numbers being given. Examples differ from "requests for the missing number in a specific quantitative relationship" in respect to the manner in which the response is given. In the latter class of exercise, the pupil is expected to memorize the response and when two numbers are given he is expected to "remember" the response. There is no calculation. In the case of examples, the pupil responds to elements of the request and builds up the response to the total situation. In addition to those given in explicit form, examples are created in solving verbal problems (see page 21) when a decision has been reached in regard to the calculations to be performed.

Response to examples. A fluent response is to be made to examples, that is, one needs to be equipped so that he can perform the specified calculation accurately and with reasonable speed. In making this response one utilizes his ability to respond to the basic and secondary number combinations.

Sometimes one is expected to be able to make a special response, that is, employ a short cut or use a calculating device such as an interest table.

V. Questions, usually implied, concerning functional relationships. A question concerning a functional relationship¹⁷ is implied in the statement of a problem. Consider the problem, "If a quart of paint covers 9 sq. yd. of floor surface, how much paint is required for the floor of a porch 12 ft. by 20 ft.?" This problem implies the question, "How is the area of the porch floor in square yards to be calculated from the dimensions 12 ft. and 20 ft.?" The problem, "Find the value

¹⁷A functional relationship is a statement of the quantitative relation between certain general quantities such as base, altitude and area of a rectangle, or the face of a note, time, rate of interest, and amount due.

of 52.3 bu. of wheat at \$1.27 per bushel," implies the question, "How is the value of a number of units (in this case 52.3 bu.) calculated when the number of units and price per unit are given?"

A search through the literature relating to the objectives of arithmetic has failed to reveal any attempt to determine the particular questions concerning functional relationship which children should learn to identify in problems and problematic situations and to answer. In an analysis of the second and third books of ten three-book series of arithmetics 333 types of questions were recognized.¹⁸ A few illustrations are given here and a suggested minimum essential list is given as Appendix B, page 90.

What calculation must be made:

To find the total, given two or more items, values, and so forth.

To find the amount, or number needed, given a magnitude and the number of times it is to be taken.

To find how many when reduction ascending is required, given a magnitude expressed in terms of two or more denominations.

To find the total price, given the number of units and the price per unit of another denomination.

To find the return percent on an investment, given the net profit or net income and amount invested.

To find the rate of profit, given the cost of goods, and the expenses and losses.

The reader should bear in mind that verbal problems implying the same question concerning a functional relation may vary greatly in form of statement. For example, the question, "How may the number of units bought or sold be calculated when the price or value per unit and the total price or value are known," is implied in each of the following problems:¹⁹

1. How many yards of silk at \$1.50 per yard can be bought for \$7.50?

2. The silk for a dress cost \$7.50. How many yards were purchased at \$1.50 per yard?

3. At \$1.50 per yard, how many yards of silk does a woman get if the amount of the purchase is \$7.50?

4. At the rate of \$1.50 per yard my bill for silk was \$7.50. How many yards were purchased?

5. How many yards of silk at \$1.50 a yard does a bill of \$7.50 represent?

6. When silk is \$1.50 a yard, a piece of silk costs \$7.50. How many yards in the piece?

7. At \$1.50 a yard how many yards of silk does a merchant sell if he receives \$7.50 for the piece?

8. Mrs. Jones purchased silk at \$1.50 a yard. The entire amount paid was \$7.50. How many yards were bought?

9. Silk was sold at \$1.50 per yard. A check for \$7.50 was given in settlement. Find the number of yards bought.

¹⁸This analysis is described on page 41 and the 333 types of questions are given in Appendix A.

¹⁹The problems of this list were suggested by analysis of several texts.

10. At \$1.50 per yard, how many yards can be bought for \$7.50?
11. A merchant sells a number of yards of silk for \$7.50. The price being \$1.50 for each yard, how many does he sell?
12. I invested \$7.50 in silk at \$1.50 per yard. How many yards did I buy?
13. When silk is \$1.50 per yard, how many yards can be bought for \$7.50?
14. When silk is sold for \$1.50 for each yard, what quantity can be bought for \$7.50?
15. At the rate of \$1.50 per yard, how many yards can be bought for \$7.50?
16. Silk is selling for \$1.50 per yard, how many yards should be sold for \$7.50?
17. At a cost of \$1.50 a yard, how many yards can be bought for \$7.50?
18. Silk was bought at a cost of \$1.50 per yard. At that rate, how many yards can be bought for \$7.50?
19. At \$1.50 a yard a piece of silk cost \$7.50. How many yards in the piece?
20. How many yards of silk at \$1.50 can I buy for \$7.50?
21. \$7.50 was paid for silk at \$1.50 per yard. How many yards were bought?
22. Find the number of yards; cost \$7.50. Price per yard \$1.50.
23. The cost of a piece of cloth is \$7.50 and the cost per yard \$1.50. How many yards are there in the piece?
24. A woman paid \$7.50 for a piece of silk that cost her \$1.50 per yard. How many yards were there in the piece?
25. A woman had \$7.50 and bought silk at \$1.50 a yard. How many yards did she buy?
26. A quantity of silk at \$1.50 per yard cost \$7.50. What was the quantity?
27. Silk is \$1.50 a yard and I bought \$7.50 worth today. How many yards did I buy?
28. A woman's bill for silk was \$7.50. If each yard cost \$1.50, how many yards were bought?

This list does not exhaust the types of statements of one-step problems which imply this question. It is also implied in combination with other questions in many problems involving two or more steps. However, the list illustrates something of the variety of situations (problem statements) to which the response "divide the total price (cost, value, etc.) by the price (cost, value, etc.) per unit and the quotient will be the number of units" is to be given.

Response to be given to questions concerning functional relationships. The type of response to be given to questions concerning general quantitative relationships is described in the preceding paragraph. The reader should note that the answer specifies certain calculations to be made.

VI. Verbal problems. Verbal problems have been referred to in the preceding pages but when they constitute occasions for manufacturing a response by reflective thinking rather than recalling a ready-made response, that is, when they are "new" and are really problems for the pupil, they justify recognition as an additional type of situation for which arithmetical equipment is needed. It is not possible to give an objective definition of the line of demarcation between the problems to which one responds by reflective thinking and the "problems" that

he "solves" by recalling a ready-made response. Almost any problem may come under the second type provided a person encounters it or very similar exercises sufficiently frequently so that the bond connecting the required response with the situation represented by the verbal statement of the problem has become fixed. When this happens the problem ceases to be a "problem" for the person in question, that is, it is not a situation requiring reflective thinking. For example, a seventh-grade pupil may think reflectively in solving an interest problem but a banker would respond to it in much the same way as the pupil responds to a request to multiply 846 by 52.

A verbal problem in the sense the term is used here is a *new* situation, that is, one for which the person does not have a ready-made response. Thus when we state that one of the objectives of instruction in arithmetic is to equip the pupils to solve verbal problems, we mean that they are to be equipped to respond satisfactorily to situations to which they have not responded previously, that is, to answer questions they have not answered in their study of arithmetic.

A *new* situation is not necessarily new in all its elements. In fact the opposite is usually true. A new problem will usually involve many familiar words and phrases. The implied questions relative to general quantitative relations will usually be familiar. The total situation, however, is new either because unfamiliar elements are introduced or because familiar elements appear in a new combination.

Response to problematic situations. As indicated in the preceding paragraphs, the nature of the response one makes is the distinguishing characteristic of a problematic situation. The response is complex. In so far as the situation is familiar, the elements of the response belong under other types of situations. Meanings are connected with words and symbols; the implied question concerning a functional relationship is identified and answered; denominate number facts are recalled; numbers are read and copied. However, the response cannot be adequately described by enumerating the responses to such elements. Reflective thinking is involved. It should be noted that the total response to a verbal problem includes the determination of the calculations to be performed plus the response to the example formulated. Reflective thinking is involved in only the first phase of the total response.

VII. Informational questions about business and social activities. Adults find occasion to answer a number of informational questions relating to such activities as banking, transportation, transmitting money, taxation, insurance, manufacturing, construction, and so forth. The following are typical: How is money transmitted? What is a promisory

note? What is a sight draft? How does a city secure funds for paving streets? How are taxes levied? What is board measure? What is overhead? How is postage computed on parcels? What conditions affect fire insurance rates? The questions may be asked in explicit terms but frequently they are implied in a general request or need.

The range of such questions for which instruction in arithmetic is expected to engender equipment has not been determined but it is obvious that other school subjects, especially geography and civics, must assume some of the responsibility for equipping pupils to answer informational questions relating to business and social activities.

Response given to informational questions relating to business and social activities. The general nature of the response to informational questions relating to business and social activities is implied by the illustrative questions in the preceding paragraph. However, it may be noted that usually precise and definite answers are required.

VIII. "Practical experiences." Under the head of "practical experiences" we group a number of types of situations such as (1) United States currency and other collections of objects to be counted, (2) magnitudes to be estimated or measured in terms of some unit, (3) business forms (sales slips, checks, money orders, etc.), catalogue lists, proposals for bond issues, newspaper quotations, and so forth to be comprehended, (4) situations in which arithmetical problems are to be identified and formulated.

The engendering of the arithmetical equipment required for responding to the situations enumerated under the head of "practical experiences" represents important objectives. The need for counting objects, estimating or measuring magnitudes, and comprehending business forms is generally recognized but the need for identifying and formulating the arithmetical problems arising in practical situations is even more important. With few exceptions adults seldom need to solve a verbal problem stated by another person. Their problems are encountered in their "practical experiences" and before the solution is begun the problem must be formulated, at least mentally.

General patterns of conduct as objectives in arithmetical instruction. As stated on page 10 a general pattern of conduct does not provide a response to a particular situation but it exercises a general control of one's responses to many situations, the range depending upon the extent of the generalization of the pattern.

Accuracy or the "habit of accuracy" is usually listed as an objective of arithmetical instruction but it is different from the specific habits

which function in performing calculations. The latter designate definite responses to definite situations. A "habit of accuracy" is a general pattern of conduct which controls responses to a variety of situations which in this case are calculations. This control may result in performing the calculation a second time, checking, inspecting the work for errors, judging the answer with respect to reasonableness, and the like. A person who has attained the "habit of accuracy" tends to give one or more of these responses to any calculation situation. The presence of the word "habit" indicates that the response always tends to be made and is made skillfully. Much the same idea is expressed by the statement that a person who possesses the "habit of accuracy" knows what to do in order to attain accuracy and how to do it, and derives satisfaction from doing what is necessary.

Other general patterns of conduct listed among the objectives of arithmetic are neatness, honesty, initiative and resourcefulness in solving problems, perseverance, and systematic procedure. A general pattern of conduct which may be designated as a "problem solving attitude" is implied in some of the statements of the aim of arithmetic. Its central element appears to be the belief that the way to respond to a new situation, that is, a problem, is to ascertain what is known about it and precisely what question or questions are to be answered, and then to focus one's resources upon the problem in an attempt to manufacture a response by formulating solutions (hypotheses) and testing them until a satisfactory one is found. Usually there is coupled with this belief, confidence in one's own ability to solve the problem. The absence of these phases of a "problem solving attitude" is evidenced when a pupil searches in his text for the solution of a similar problem or restricts his efforts to recalling the solution of a similar problem.

Another significant phase of the "problem solving attitude" is involved in the pupil's concept of what it means to solve a problem. One point of view is that to solve a problem is to get the answer given in the text or one that will be accepted by the teacher. The "problem solving attitude" requires that one think of the solving of a problem as a case of reflective thinking in which the fundamental objective is to conform to the requirements of good thinking.

Summary. Although the preceding discussion of the objectives of arithmetic has filled several pages, it has doubtless been apparent to the reader that the items of mental equipment (specific habits, knowledge and general patterns of conduct) to be engendered by the instruction in

arithmetic have not been specified at all completely.²⁰ However, the enumeration of eight types of situations to which pupils are to be equipped to respond and the consideration of general patterns of conduct should lead the reader to attach more meaning to the first statement of the purpose of instruction in arithmetic (see page 7). The teaching of arithmetic is expected to engender the specific habits, knowledge, and general patterns of conduct needed for responding in a satisfactory way to the following general classes of situations:

- I. Number symbols.
- II. Other arithmetical symbols and technical terms.
- III. Two or more numbers quantitatively related with one missing.
- IV. Examples.
- V. Questions (usually implied) concerning functional relationships.
- VI. Verbal problems.
- VII. Informational questions about business and social activities.
- VIII. Practical experiences: (1) collections of objects to be counted, (2) magnitudes to be estimated in terms of some unit, (3) business forms, catalogue lists, newspaper quotations, and so forth to be comprehended, (4) situations in which arithmetical problems are to be identified and formulated.

²⁰The situations for which ability to respond should be engendered have been described only in terms of types and no attempt has been made to specify the quality of the several abilities.

CHAPTER II

THE PROCESSES OF LEARNING AND TEACHING

The discussion of the objectives of arithmetic in Chapter I furnishes a description of what pupils should learn during their study of this subject in the elementary school. The problem of this chapter is to describe certain phases of the learning process and the teaching procedures that are essential to the attainment of these objectives.

Learning an active process. In discussing the work of the teacher, we commonly use verbs such as "impart," "communicate," "present," "explain," and "instruct," which not infrequently appear to imply that in the process of educating children the teacher transmits specific habits, items of knowledge or general patterns of conduct to the pupil whose mind may be receptive or even eager to receive or may be indifferent or hostile. Although no one who is informed in regard to modern psychology would support such a theory of learning, the reading of current educational literature and the observation of classroom procedures suggest that many teachers, in planning lessons and in conducting recitations, do assume that their function is to transmit ideas, facts, rules, and other items of knowledge to their pupils.

The statement, "learning is an active process," is commonplace but its significance is far-reaching. What a child learns is the product of his own activity, physical, mental, and emotional. A child who is not active does not learn. In order to learn the multiplication combinations a child must engage in certain types of activity;¹ in no other way can he acquire the necessary specific habits (fixed associations).

Assignment of exercises required as a basis for attaining arithmetical objectives. Acceptance of the thesis, that learning is an active process and that the acquiring of certain abilities requires participation in certain types of learning activities, raises the question: What means must the teacher employ to secure the pupil activity that will lead to the attainment of the objectives described in Chapter I? A child learns as the result of his activity outside of the school such as playing games; doing errands for his parents, including the making of purchases; reading newspapers, magazines and books; constructing toys, playhouses

¹The reader should not interpret this statement to mean that all pupils must go through the same activities. The activities of one pupil engaged in learning the multiplication combinations may differ in certain respects from those of another pupil working toward the same end but the activities of the two pupils will have certain common characteristics. For example, in this case both will involve repetition.

and the like; being a member of an organization such as the Boy Scouts or Campfire Girls; observing the activities of adults, and the like. However, the establishment of schools is evidence of the recognition of the fact that the abilities resulting from participation in such activities during the period of childhood would seldom if ever constitute adequate equipment for meeting the demands of adult life. If our schools were abolished, many necessary abilities would not be acquired and the quality of others would be unsatisfactory. Prior to the inclusion of arithmetic in the curriculum of the "public schools," very few children acquired ability to "cipher" and those who learned arithmetic did so as the result of attending a special school in which it was taught.

Participation in efficient educative activities in school is not secured by the teacher making the direct request, "Be active" or "Do something." In response to such commands or requests the pupils may become active. In fact children are by nature inclined to be active and unless restrained will "do something," but "spontaneous" activity is very unlikely to contribute greatly to their mental equipment, at least in the field of arithmetic. Observation of the teaching of arithmetic justifies the statement that most of the educative activity in this field is in response to definite exercises² assigned by the teacher.

Those who believe in the project method will probably take issue with the thesis of the preceding paragraph. It is true that needs which furnish a basis for efficient educative activities do arise in the attempts of pupils to realize their own immediate purposes both in and out of school. It is also true that the number of such needs can be increased by skillful encouragement and manipulation of conditions by the teacher, but it does not appear that in general the project method can be depended upon to stimulate the learning activities necessary to produce the mental equipment specified by the objectives of arithmetic. In most teaching situations there must be explicit assignment of exercises which will function as the basis for much, and in many instances practically all, of the educative activity in the field of arithmetic.³

Types of learning activities resulting in the acquisition of specific habits and knowledge. Before consideration is given to the types of exercises assigned by teachers of arithmetic, it will be helpful to note the types of learning activities in which children engage in acquiring

²The types of exercises employed in arithmetic will be considered later (see page 33).

³For a more complete discussion of the "project method" versus the "assignment method" see:

MONROE, WALTER S. "Projects and the project method." University of Illinois Bulletin, Vol. 23, No. 30, Bureau of Educational Research Circular No. 43. Urbana: University of Illinois, 1926. 20 p.

specific habits and knowledge. In attempting to analyze mental activity, psychologists have identified such processes as sensation, perception, conception, imagination, memory, association, analysis, generalization, and reasoning. Recognition of these mental processes is helpful for certain purposes but a somewhat different analysis of learning appears to provide a more practical basis for considering the technique of teaching. The "types of learning activity" enumerated in the following paragraphs represent a pedagogical rather than a scientific analysis of the learning process.

1. Direct or perceptual experiencing occurs in learning arithmetic when a pupil counts objects, measures the length of the room, handles weights or money, steps off a distance, and the like. In direct experiencing there is perception and hence the functioning of one or more of the sense organs. Perceptual experiences are required as a basis for the other rubrics of learning activity. Much of the necessary experiencing will take place outside of the school but when additional experiencing is required the teacher must provide opportunity for this type of learning activity. Perceptual experiencing occurs in measuring and counting objects, dramatizing adult activities, visiting business concerns such as banks, grocery stores, department stores, and the like.

2. Vicarious experiencing occurs when one listens to or reads an account of the perceptual experiences of another, provided the listener or reader comprehends the terms used in the description. For example, a pupil may experience vicariously or second-hand, without visiting and observing its activities, the operation of a building and loan association. When a person observes an activity such as an athletic contest his experience is a combination of direct and vicarious. His playing of the game is vicarious but his seeing of the players and the spectators is direct. A foundation of direct or first-hand experience is an essential prerequisite for vicarious experiencing.

3. Generalizing experience⁴ is used as a name for analyzing, comparing, organizing, and abstracting experiences, both direct and vicarious. The products of these activities are called concepts, rules, principles, generalizations, and abstractions. Words like sum, multiplication, interest, premium, volume, and fraction represent concepts. They are sometimes called abstract and general meanings.

4. Comprehending the products of thought expressed in terms of words, phrases or sentences designates a type of learning activity which in some respects is the reverse of generalizing experience. In the

⁴"Inductive development" has also been used as a name for this type of activity.

course of the history of the race a number of terms have been developed for use in describing arithmetical calculations and in stating problems. A pupil encounters such words as addition, multiplication, interest, premium, numerator, percent, and the like. Before he has learned from his experiencing the meaning they represent he faces the necessity of comprehending or understanding the products of the thinking of other persons. A similar statement can be made with reference to rules and principles.

5. Using one's knowledge in manufacturing a response to a new situation⁵ is commonly called "problem solving" or reflective thinking. These terms, however, are used somewhat carelessly and for this reason the writer has chosen a descriptive phrase which explicitly specifies that the learner is engaged in responding to a new situation. It is generally assumed that reflective thinking occurs when a pupil responds to a verbal problem. This is not true because the pupil may remember how the problem or a similar one was solved in the text or by the teacher, or he may search his text for the solution of a similar problem. When this constitutes his activity he is not thinking reflectively, that is, he is not manufacturing a response to a new situation. He is simply searching for a ready-made response. Random guessing represents another type of activity that is not included under the caption used here. "Using knowledge in manufacturing responses to new situations" is an important type of learning activity. Its occurrence is not confined to solving verbal problems.

Thought questions that do not involve arithmetical calculations also furnish a basis for using knowledge. Whenever a student encounters a difficulty or is asked a question which he is unable to answer, he has a problem to solve. If he manufactures a solution for it he engages in reflective thinking and consequently engages in learning activity.

6. Tracing the thinking of another person by listening to an oral description of it or by reading a printed record constitutes a sixth type of learning activity. It occurs in arithmetic when a pupil listens to an explanation of the solution of a problem given by the teacher or by another pupil.

7. Expressing one's ideas is educative, particularly when attention is given to their evaluation and organization. A pupil learns by explaining the solution of a problem but the amount of learning may become almost negligible if he merely follows a definite formula, as was frequently required in the teaching of mental arithmetic. Expression of ideas also occurs in responding to thought questions.

⁵See page 22.

8. "Prolonging, repeating and intensifying one's experiences,"⁶ represents a type of learning activity which is very prominent in arithmetic. "Drill" or "practice" clearly comes under this type of learning activity. "Living over" perceptual experiences, recalling what has been read or heard, thinking through the solution of a problem, retracing an explanation given by the teacher, reconnecting a meaning with an abstract or general term, and the like are also illustrations of "repeating experiences."

9. **Learning activities resulting in the acquisition of general patterns of conduct.** A description of the activities that result in the acquisition of general patterns of conduct is difficult, but it appears that the production of this class of outcomes tends to be governed by subtle factors. The specific habits necessary for responding to quantitative relationships in which one number is missing⁷ can be engendered by having the pupil engage in appropriate practice. On the other hand, the "habit of accuracy" does not result from engaging in any certain activities. Two pupils may apparently engage in the same learning activities and one will acquire the "habit of accuracy" while the other will not.

General patterns of conduct have been described as by-products which *may be* produced in the acquiring of specific habits and knowledge. This appears to be a valid description but it should be noted that the statement is "may be produced" rather than "are produced." The engendering of general patterns of conduct is incidental but not accidental. They result when the conditions are right; they do not when the necessary conditions are not secured. However, our knowledge of this phase of the learning process is not yet sufficient for us to specify in detail what conditions are necessary for the engendering of a general pattern of conduct.

Studying and reciting frequently involve a combination of types of learning activities. The preceding analysis of learning activity is not intended to imply that each type occurs separately and independently. Frequently the total activity of the pupil is a combination of two or more types. Perhaps a more accurate statement would be that the learner, in doing a school exercise, may shift from one type of activity to another. For example, in attempting to solve a verbal problem one may "trace the thinking" of the teacher by listening to an explanation

⁶This phrase is used by:

BOBBITT, FRANKLIN. *How to Make a Curriculum*. New York: Houghton Mifflin Company, 1924, p. 57. The word "experiences" in this expression has a somewhat broader meaning than on page 28.

⁷See page 18.

of certain phases of it or "comprehend the meaning of a new abstract term" in addition to thinking reflectively about the problem.

Relation between types of learning activity and rubrics of abilities. In the preceding pages we have described the types of learning activity in which children engage and the types of abilities that are produced as outcomes. Perhaps the reader has already raised the question: just how are the different types of activity related to the different kinds of ability; or more specifically, if a pupil engages in a specified kind of learning activity, what will be the nature of the resulting outcomes.

Although the relation between mental activity and the resulting outcomes cannot be stated in terms of precise laws, such as have been formulated in chemistry and physics, it is possible to state certain general laws. Any ability (specific habit, knowledge, or general pattern of conduct) may be thought of as a response connected with a stimulus and its quality depends upon the strength of the connection as well as upon the response. For example, $6 \times 7 =$ is a stimulus, and 42 the response. The strength of the connection between $6 \times 7 =$ and 42 is a very important element in a pupil's ability to respond to 6×7 .

In each of the types of learning activity there is the exercise of connections between stimuli (situation) and responses. In direct experiencing the connection is between the stimulus apprehended by means of a sense organ and resulting percept. In "problem solving" there is a sequence of stimuli and responses, both of which usually are ideas, meanings, concepts, and the like. In solving a problem a pupil manufactures a "new response" (the solution) which is connected with the problem.

Laws governing the effectiveness of learning activities. The effectiveness of any learning activity in producing mental equipment (specific habits, knowledge or general patterns of conduct) is governed by certain laws⁸ which may be stated as follows:

I. If other factors affecting learning remain unchanged, the strength of a modifiable connection between a situation and a response is strengthened as it is exercised and up to a certain limit the strength of the connection increases with the amount of exercise but not in a constant ratio.

⁸These "laws of learning" are based on formulations by:

THORNDIKE, E. L. Educational Psychology, Vol. II. New York: Teachers College, Columbia University, 1913. 452 p., or Educational Psychology, Briefer Course. New York: Teachers College, Columbia University, 1914, Part II.

GATES, ARTHUR I. Psychology for Students of Education. New York: The Macmillan Company, 1925, Chapter X.

II. Other conditions being equal, the more recent the exercise of a modifiable connection between a situation and a response, the stronger the connection is. This implies that a connection which is not exercised gradually grows weaker.

III. The effect of the exercise of a modifiable connection between a situation and a response depends upon the degree of satisfaction that accompanies or follows the activity. Other conditions being equal, when "a satisfying state of affairs" prevails the connection is strengthened; when a state of dissatisfaction or annoyance prevails, the connection is weakened.

IV. The strengthening effect of the exercise of a modifiable connection between a situation and a response depends upon the distribution of the exercise, and other things being equal the maximum effect is obtained by distributing the exercise rather than by concentrating it.

V. The subject's capacity to learn (commonly called general intelligence) contributes to the effect of the exercise of a modifiable connection between a situation and a response.

Predicting activity necessary for attainment of specific objectives.

When the teacher has formulated her immediate objectives, that is, certain specific habits, items of knowledge or general patterns of conduct to be engendered, she then must predict the learning activities in which it will be necessary for her pupils to engage in order to attain these objectives. For example, suppose the immediate objectives are the fixed associations designated as the multiplication combinations, the teacher's problem is: "In what activities must I get my pupils to engage in order to learn these fixed associations?" Our knowledge of the relations between learning activity and outcomes enables us to conclude that perceptual experiencing is necessary to provide a foundation for the concept of multiplication. There must also be generalizing of these experiences and of course many repetitions of the exercise of the connections between the products and the numbers whose products are being learned.

CHAPTER III

THE LEARNING EXERCISES OF ARITHMETIC

The problem of this chapter. It is the problem of this chapter to identify and describe the kinds of exercises which may be assigned by teachers as the basis for the activity necessary for the attainment of the objectives described in Chapter I. No attempt will be made to determine the relative effectiveness of the several types of exercises or to indicate when each should be used.

Classes of learning exercises. The analysis of learning activity in the preceding chapter might be used as a basis for a corresponding list of types of exercises but it has seemed desirable to recognize other factors than the general type of ensuing mental activity. For example, counting objects and measuring linear distance involve perceptual experiencing but they are sufficiently different to justify listing them as two separate types of exercises. The following list is not offered as a complete enumeration of the classes of requests¹ that teachers of arithmetic make of their pupils but it will serve to indicate their range.

1. Requests to count objects such as the children in the class, the windows in the classroom, marks on the blackboard or in the textbook, and the like.

2. Requests to measure magnitudes such as length of desk or room, the water in a pail, and the like.

3. Requests to estimate physical magnitudes.

4. Projects and construction exercises including those involving cooking, sewing, gardening, and the like. These involve implied requests to detect and formulate needs for measuring, computing, keeping records, and the like.

5. Games involving quantitative activities such as keeping score.

6. Requests to visit such places as a retail store or bank in order to observe adult activities.

7. Requests to describe perceptual experiences relating to arithmetic.

¹An analysis of two of the classes, examples and verbal problems, is given, beginning p. 35.

8. Accounts of experiences or other descriptions to be listened to or read.
9. Requests to generalize experiences. (Usually these requests are not direct. See page 28.)
10. Dramatization of an adult activity such as farming, manufacturing, banking, or keeping store.
11. Dictation exercises in which numbers and other arithmetical symbols are to be written.
12. Requests to copy numbers and other arithmetical symbols from the text or blackboard.
13. Requests to read orally numbers and other arithmetical symbols.
14. Requests to repeat orally or to write certain groups of symbols or facts. This includes counting by 2's, 3's, and so forth as well as groups commonly designated as "tables."
15. Requests to memorize, that is, to repeat lists of facts, technical terms, abbreviations and the like without specific exercises calling for repetitions.
16. Requests, both oral and written, for the missing number in a specific quantitative relationship, such as $7 + 5 = ?$, $27 + 6 = ?$, $24 \div 4 = ?$, $47 \div 9 = ?$, $.12\frac{1}{2} = ?$, $1 \text{ yd.} = ? \text{ ft.}$
17. Explicit requests to perform specified calculations, commonly called "examples."² (For types of examples see page 35.)
18. Verbal problems.³ (For types of verbal problems see page 41.)
19. Requests to explain performed calculations or solutions of problems.
20. Requests to read or listen to an explanation.
21. Requests to check calculations.
22. Requests to inspect and verify solutions of problems.
23. Fact questions other than requests to supply the missing number in a quantitative relationship. (Questions concerning functional relationships would be included here when the pupil does not find it necessary to think reflectively in answering them.)

²For definition of example see page 19.

³Usually a verbal problem is to be solved but a pupil may be requested to estimate the answer.

24. Thought questions.⁴ (These include questions concerning general quantitative relations and problems without numbers.)

25. Requests to read and comprehend descriptions, definitions, rules and abstract terms.⁵

26. Requests to read, or reproduce, business forms such as checks, notes, sales slips, and so forth.

27. Requests to collect or formulate problems.

28. Requests to collect quantitative information such as prices or other items in regard to business practices.

29. Graphs to be read.

30. Groups of data to be represented graphically.

31. Requests to use tables and other calculating devices.

Variations within the classes of learning exercises. Each of the classes in the preceding list includes learning exercises that differ in certain respects. Some of these differences are significant but others are not. The exercises included in the first class, "Requests to count objects," differ with respect to the kind of objects to be counted. Such a difference has little or no significance because the counting of objects is essentially the same as a learning activity regardless of the nature of the objects counted.⁶ On the other hand, in the second class the measurement of linear distance differs in a significant way from the measurement of mass because the instruments and units are different.

"Specific requests to perform certain calculations" (examples) and "verbal problems" represent very complex classes of learning exercises. Since "examples" and "problems" are used extensively as bases of learning activity in the field of arithmetic, it will be helpful to note the types of exercises included in each of these classes.

Types of examples. The term "example" is used as the name for an explicit request to add, subtract, multiply, or divide,⁷ the numbers being given. The example may call for two or more of these operations to be performed but in all cases the request is explicit. The request for the calculation may be expressed in terms of symbols, such

⁴For a general discussion of types of thought questions, see:

MONROE, WALTER S., and CARTER, RALPH E. "The use of different types of thought questions in secondary schools and their relative difficulty for students." University of Illinois Bulletin, Vol. 20, No. 34. Bureau of Educational Research Bulletin No. 14. Urbana: University of Illinois, 1923. 26 p.

⁵Formulae may be added.

⁶This statement is not intended to imply that a pupil's activity is essentially the same in all cases. The point made is that, other things being equal, the nature of the objects counted is not significant.

⁷The extraction of roots may be added as a fifth calculation process.

as $694 \div 27 = ?$, $37 \overline{)848}$, or technical terms may be used, as "Find the product of 87 and 64," "Divide 694 by 27."

When considered as learning exercises, it is obvious that examples which differ in any way do not afford the basis for identical mental activities. The connections exercised by responding to 646×23 are different from those exercised by responding to 646×67 . However, the difference is dissimilar to that existing between the responses to "subtract 746 from 9286" and "divide $18\frac{3}{4}$ by $1\frac{5}{4}$." In the case of the responses to 646×23 and 646×67 , we may say that they are similar in the sense that each involves the exercises of multiplication and addition combinations. This condition of similarity is expressed by saying all examples calling for the multiplication of a three-place integer by a two-place multiplier constitute a type. Examples such as 387×6 provide a sufficiently different learning activity to justify recognition as another type. A request to multiply 412×4 constitutes a third type since no carrying is involved.

Recognition of differences of the kind illustrated in the preceding paragraph raises the the question, "What are the significant types of arithmetical examples?" Those involving only one calculation process fall naturally into four general groups; (1) addition, (2) subtraction, (3) multiplication, and (4) division. Within each of these groups three subdivisions are created by the three types of numbers; integers, common fractions and decimals. Each of these twelve divisions obviously includes examples that differ in certain respects. Some of these differences have been recognized in arithmetic texts for many years by explicit "cases" such as "short multiplication," "subtraction with borrowing" and the like. However, the "cases" usually mentioned do not appear to constitute a complete enumeration of the types of examples. In addition a long column of figures (12 to 15) appears to constitute a different type of example from that furnished by a short column of figures (3 to 5). In multiplication the presence of zeros in the multiplier appears to create at least one separate type of example and possibly two.

• We have little experimental evidence concerning the differences that must exist between two examples in order to require that they be listed as belonging to separate types. The following lists are intended to be conservative. Several of the types include examples which differ in certain respects and it may be that the differences are sufficiently significant to justify the recognition of subordinate types. However, the enumeration of the types given here will serve to show the general character of the learning exercises which are commonly called examples.

I. ADDITION OF INTEGERS⁸

1. Short column addition,⁹ 3 to 5 addends:

$$\begin{array}{r} 8 \quad 3 \quad 9 \\ 4 \quad 2 \quad 4 \\ 7 \quad 4 \quad 5 \\ \hline 8 \\ 3 \\ \hline \end{array}$$

2. Long column addition. (Two or more sub-types may be recognized by making divisions on basis of length.) Frequently columns of more than 7 to 9 addends constitute a situation different from that furnished by an example of 5 to 7 addends because the "span of attention" is increased beyond the normal length.

3. Addition with carrying.

4. Addition of numbers of different lengths.

II. SUBTRACTION OF INTEGERS

1. Subtraction of a number of one digit from a number of two digits. (Such subtractions may be considered as additional combinations corresponding to the "higher decade combinations" in addition.)

2. Subtraction of numbers of two or more digits involving "borrowing," but no zero in either subtrahend or minuend.

3. Subtraction of numbers of two or more digits involving "borrowing" and one or more zeros in the minuend.

$$\begin{array}{r} 840 \\ \hline 73 \end{array} \quad \begin{array}{r} 507 \\ \hline 184 \end{array} \quad \begin{array}{r} 1000 \\ \hline 63 \end{array} \quad \begin{array}{r} 602 \\ \hline 276 \end{array}$$

4. Subtraction of numbers of three or more digits with at least one zero in the subtrahend.

$$\begin{array}{r} 896 \\ \hline 170 \end{array} \quad \begin{array}{r} 383 \\ \hline 207 \end{array}$$

III. MULTIPLICATION OF INTEGERS

1. Short multiplication with carrying.¹⁰

2. Long multiplication without carrying.

3. Long multiplication with carrying.

4. Multiplications involving one or more zeros in multiplicand.

$$\begin{array}{r} 8350 \\ \hline 92 \end{array} \quad \begin{array}{r} 705 \\ \hline 37 \end{array}$$

5. Multiplications involving one or more zeros in multiplier.

$$\begin{array}{r} 4736 \\ \hline 805 \end{array} \quad \begin{array}{r} 845 \\ \hline 30 \end{array}$$

IV. DIVISION OF ONE INTEGER BY ANOTHER

1. Short division: divisor 1 to 9, no zeros in quotient, with or without remainder.

⁸An example may be expressed in two or more ways:

$$\begin{array}{r} 18 \\ 33 \\ 187 \\ \hline \end{array} \quad \begin{array}{l} 18 + 33 + 187 = \quad . \\ \text{Add } 18, 33, \text{ and } 187. \\ \text{Find the sum of } 18, 33, \text{ and } 187. \end{array}$$

Such variations in form are not considered here.

⁹As the term has been used in the preceding pages (see page 19) the basic and secondary combinations do not constitute examples.

¹⁰Short multiplication without carrying might be listed as a separate type but such examples are essentially only groups of fundamental combinations.

2. Short division: divisor 1 to 9, one or more zeros in quotient, with or without remainder.

3. Long division:¹¹ trial quotient true quotient, no zeros in quotient, no carrying in multiplications, no borrowing in subtractions and no remainder. (This is the simplest type of long division example.)

4. Long division: trial quotient true quotient, no zeros in quotient, and no remainder. (This differs from the preceding by permitting carrying in the multiplications and borrowing in the subtractions.)

5. Long division: trial quotient true quotient, no zeros in quotient but with remainder.

6. Long division: trial quotient not true quotient, no zeros in quotient and no remainder.

7. Long division: trial quotient not true quotient, no zeros in quotient, but with remainder.

8. Long division: zeros in quotient. (This may be considered a composite of several types. When a zero occupies units place in a quotient the example is probably different from those in which it appears in an interior position. Differentiations might also be made in respect to the trial quotient and the remainder.)

V. ADDITION OF FRACTIONS¹²

1. Addition of two or more fractions with common denominators, the sum being non-reducible, that is, in its lowest terms and less than unity.

2. Addition of two or more fractions with common denominators, the sum being reducible.

3. Addition of two or more fractions, the denominators not being common. (The examples under this type may be divided according to the reducible quality of the sum.)

4. Addition of mixed numbers.

5. Addition of an integer and a fraction, the sum to be expressed as an improper fraction.

VI. SUBTRACTION OF FRACTIONS

1. Subtraction of fractions having common denominators.¹³

2. Subtraction of fractions not having common denominators.

3. Subtraction of a fraction from a mixed number, requiring borrowing.

4. Subtraction of one mixed number from another, not requiring borrowing.

5. Subtraction of one mixed number from another, requiring borrowing.

6. Subtraction of a fraction from an integer.

VII. MULTIPLICATION OF FRACTIONS

1. Multiplication of an integer greater than unity by a unit fraction such as $\frac{1}{2}$ or $\frac{1}{3}$.

2. Multiplication of an integer greater than unity by other proper fractions.

3. Multiplication of one unit fraction by another unit fraction.

4. Multiplication of two fractions neither of which is a unit fraction. Product may be either reducible or non-reducible.

¹¹The possibility of a large number of types of long division examples is apparent from the fact that five conditions are specified in defining this type. The list given here includes only what appears to be the most significant types of long division examples.

¹²For a much more elaborate list of types of examples in the addition of fractions see:

KALLOM, ARTHUR W. "Analysis of and testing in common fractions," *Journal of Educational Research*, 1:177-92, March, 1920.

¹³The character of the difference is not considered a differentiating factor. If this were done additional types would be found.

5. Multiplication of a mixed number and a fraction. Fractional product may be reducible or non-reducible.

6. Multiplication of a mixed number by an integer.

7. Multiplication of two mixed numbers.

VIII. DIVISION OF FRACTIONS¹⁴

As stated on page 36 the list of types presented here is not intended to include all possible ones and some of those given obviously include examples which differ in certain respects.

1. Division of an integer by a unit fraction.

2. Division of an integer by other proper fractions.

3. Division of one fraction by another.

4. Division of a fraction by an integer.

5. Division of a fraction by a mixed number.

6. Division of a mixed number by a fraction.

7. Division of a mixed number by a mixed number.

8. Division of a mixed number by an integer.

9. Division of an integer by a larger integer.

IX. ADDITION OF DECIMALS¹⁵

1. Addends form addition example with right hand margin even and no zeros to the left of the last significant figure.¹⁶

.6	5.08	.4876
.5	1.26	.8428
.8	7.31	.9371
<u>.3</u>	<u>12.83</u>	<u>.8476</u>

2. Addends form addition example with right hand margin uneven but no zeros to the left of the last significant figure.

.6	17.
.346	.8942
.15	.327
<u>.3</u>	<u>1.25</u>

¹⁴It would be possible to increase the number of example types under this and the other groups by listing all of the possible combinations of the conditions affecting the example. In the case of the division of fractions, the dividend may be (1) a unit fraction, (2) other proper fractions, (3) an improper fraction, (4) a mixed number or (5) an integer. The same possibilities exist in the case of the divisor. The quotient furnishes another basis of differentiation. It may be (1) a proper fraction in lowest terms, (2) an improper fraction in lowest terms, (3) a proper fraction but not in lowest terms, (4) an improper fraction not in lowest terms, or (5) an integer. An indication of the number of possible types of examples under division of fractions is given in an article by:

Knight, F. B. "A note on the organization of drill work," *Journal of Educational Psychology*, 16:108-13, February, 1925. In this article the division of fractions is divided into 55 units of skill.

¹⁵The significant differences between addition of integers and addition of decimals have not been determined. It appears to be a reasonable hypothesis that the differences are confined to (1) placement of the decimal point in the sum, (2) the possible uneven right hand side of the addition example, and (3) the possible presence of zeros between the decimal point and the first significant figure of the addends. It does not seem that the presence of integers in the addends either separately or in combination with a decimal should constitute a significant characteristic. A similar statement may be made in the case of subtraction.

¹⁶The "last" figure is the one farthest left.

3. Addends have zeros to the left of the last significant figure.

$$\begin{array}{r} .05 \\ .0082 \\ .075 \\ .04 \\ \hline \end{array} \quad \begin{array}{r} .05 \\ .06 \\ .04 \\ .03 \\ \hline \end{array}$$

X. SUBTRACTION OF DECIMALS

1. The right hand figure of the subtrahend is written under the right hand figure of the minuend and no zeros to the left of the last significant figure in either decimal.

$$\begin{array}{r} 12.5 \\ 8.2 \\ \hline \end{array} \quad \begin{array}{r} .75 \\ .42 \\ \hline \end{array}$$

2. The right hand figure of the subtrahend farther removed from the decimal point than the right hand figure of the minuend, but no zeros to the left of the last significant figure in either decimal.

$$\begin{array}{r} 1. \\ .25 \\ \hline \end{array} \quad \begin{array}{r} .75 \\ .125 \\ \hline \end{array}$$

3. The right figure of the minuend farther removed from the decimal point than the right hand figure of the subtrahend, but no zeros to the left of the last significant figure in either decimal.

$$\begin{array}{r} .875 \\ .5 \\ \hline \end{array}$$

4. Zeros to the left of the last significant figure in at least one of the decimals.

$$\begin{array}{r} .0025 \\ .0042 \\ \hline \end{array} \quad \begin{array}{r} .875 \\ .012 \\ \hline \end{array}$$

XI. MULTIPLICATION OF DECIMALS

1. Multiplication of a decimal by an integer.

$$\begin{array}{r} .75 \\ 5 \\ \hline \end{array} \quad \begin{array}{r} .875 \\ 64 \\ \hline \end{array} \quad \begin{array}{r} 1.25 \\ 8 \\ \hline \end{array}$$

2. Multiplication of an integer by a decimal.

$$\begin{array}{r} 845 \\ .06 \\ \hline \end{array} \quad \begin{array}{r} 950 \\ .7 \\ \hline \end{array} \quad \begin{array}{r} 837 \\ 1.5 \\ \hline \end{array}$$

3. Multiplication of a decimal by a decimal.

XII. DIVISION OF DECIMALS¹⁷

1. Division of a decimal by an integer with no remainder.

2. Division of a decimal by an integer with a remainder that may be completely expressed by additional decimal places in the quotient.

3. Division of a decimal by an integer with a remainder, quotient to be carried to a specific number of decimal places.

4. Division of an integer by a decimal. (It is possible that sub-types are formed by divisors such as .6, .06, .0006, 1.06.)

5. Division of one integer by another with a remainder, quotient to be carried to a specific number of decimal places.

¹⁷See:

MONROE, WALTER S. "The ability to place the decimal point in division." *Elementary School Journal*. 18:287-93, December, 1917. This investigation indicated that many pupils do not place the decimal point in division by applying a general rule but use a special rule or device for different cases. If a special rule or device were employed for all possible combinations of dividend and divisor, the number of types of examples would be large even if the quantities were restricted to relatively few decimal places. Only a few of the more significant types are given here.

6. Division of one decimal by another when the number of decimal places in the dividend equals or exceeds those of the divisor. with no remainder. ("No remainder" implies that if the decimal point were removed from both dividend and divisor, the former would be larger than the latter.)

7. Same as the preceding except with a remainder.

8. Division of one decimal by another when the number of decimal places in the dividend is less than those in the divisor.

Types of problems. When we examine the problem lists in current arithmetics, we find a conspicuous lack of uniformity in the captions by which these lists are designated in different texts. Formerly, most of the problems given in an arithmetic were listed under such captions as: "Rule of three, direct," "Rule of three, inverse," "Partnership," "Alligation," "Barter," "Practice," "Profit and Loss," "Trade discount," "True discount," "Partial payments," "Exchange," and so forth. Changes in business practices and in the activities of adults apart from the carrying on of business have created new "applications" of arithmetic. Many of the titles formerly used as captions for problem lists have been discarded and new ones substituted. The result is that at the present time we have no generally recognized plan for classifying the problems of arithmetic.

The buying and selling of commodities, borrowing money, constructing houses and other buildings, insuring property, carrying on a business, and the like create many arithmetical problems. This suggests that the sources of problems be used as a basis for their classification¹⁸ but such a plan will not give groups which approximate homogeneity with respect to the activity required in solving the problems. Furthermore, an examination of the problems in our arithmetics will reveal a number of problems whose source is not easily identified. In some cases the problem does not appear to be connected with any particular activity or the suggested adult activity might be changed without affecting the problem. The following are typical:

1. "A pail of milk holding 2 gallons is to be poured into quart bottles. How many bottles will be needed?"

2. "Henry caught three fish. The first weighed 12 ounces, the second 10 ounces, and the third 15 ounces. What was the total weight of the three?"

3. "On a vacation trip Robert walked $6\frac{1}{2}$ miles the first day, 7 miles the second day, and $5\frac{3}{4}$ miles the third day. Find the total distance traveled."

4. "The length of an iron rod was $95\frac{15}{16}$ inches. After it was heated its length was found to be $96\frac{7}{32}$ inches. How much was the length increased by heating?"

¹⁸The writer has employed this method of analysis. See:

MONROE, WALTER S. "A preliminary report of an investigation of the economy of time in arithmetic." Second Report of the Committee on Minimum Essentials in Elementary School Subjects. Sixteenth Yearbook of the National Society for the Study of Education, Part I. Bloomington, Illinois: Public School Publishing Company, 1917, p. 111-27.

In the first problem "milk" could be changed to "water," "syrup," or any other liquid without changing the problem. Furthermore, "bottles" could be changed to "cans" or "jars." Hence, there is no significant connection between the problem and any adult activity. A similar conclusion applies to the other problems.

Thus it appears that there are two general classes of problems: A, Operation Problems, those not identified with a particular activity or identified with an activity that does introduce a technical terminology peculiar to that activity; B, Activity Problems, those identified with a definite activity of children or adults which introduces a technical terminology.

Within each of these two general classes of problems,¹⁹ a further differentiation may be based upon the implied question concerning the functional relationship. (See page 19.) All problems that ask the same question may be considered to form a problem type which may be described by designating the quantities given and the one to be found. The question concerning functional relationship is: What calculations are to be performed upon the given quantities in order to obtain the one to be found?

An elaborate study of the problems provided by texts²⁰ resulted in the identification of 52 problem types in the field of "operation problems" and 281 problem types under "activity problems."²¹ Descriptions of representative problem types are given in the following pages. A complete list of all problem types is printed in Appendix A. See also pages 90-92.

A1 To find totals by addition, given two or more items, values, etc.

A3 To find the amount, or number needed, by multiplication, given a magnitude and the number of times it is to be taken.

¹⁹A verbal problem in arithmetic is a description of a quantitative situation or condition plus a question that usually requires a numerical answer. The solving of this requires the determination of the calculations to be made in order to obtain the answer. The basis of the determination of the calculations to be performed in the solving of a problem is the general quantitative relation which connects the quantities of the problem. For example, consider this problem: "An agent sells goods on a commission of 10 percent. How much does he remit to his principal for sales amounting to \$1150?"

The quantities of this problem, proceeds (amount remitted to principal), rate of commission and amount of sales are related as follows: $\text{Proceeds} = \text{amount of sales} - \text{the product of amount of sales and rate of commission}$. In order to solve this problem rationally, that is, by reasoning, it is necessary that one answer the question, "How is the amount to be remitted to the principal calculated from the amount of sales and the rate of commission?"

²⁰See page 48 for a description of this study. The "problem types" are used in explaining the process of solving verbal problems. See page 21.

²¹A few types of learning exercises are included that do not require calculation. See Appendix A, A24, A25, B5f, B5h, B5i, B5j, B5k, B5l.

- A5 To find how many times a stated quantity is contained in a given magnitude, given the quantity and the magnitude.
- A6 To find how many when reduction ascending is required, given
 a. a magnitude expressed in terms of a single denomination.
 b. a magnitude expressed in terms of two or more denominations.
- A8 To find a dimension, given the area of a rectangle and one side.
- A13 To find a difference, given denominate numbers of different denominations.
- A15 To find the ratio of one number to another, given the two numbers.
- A16 To find a part of a number, given the ratio of the part to the number and the number. (The fraction may be in terms of fractions or decimals.)
- B1 Buying and selling,²² simple cases.
- a. To find the total price:²³
 1. given the number of units and price²⁴ per unit.
 2. given the number of units and the price per unit of another denomination.
 - b. To find the number of units:
 1. given the total price and price per unit.
 2. given the total price and the price per unit in another denomination.
 3. received in exchange of commodities, given an amount of each commodity and the unit for each.
 4. given the price per unit of each of two commodities, the total price of both, and the ratio of the number of units of one to the number of units of the other.
 5. given the margin²⁵ per unit and the total margin.
 - c. To find the price per unit:
 1. given the total price and the number of units.
 2. given the total price and the number of units in another denomination.
 3. in exchange of commodities, given the number of units of each commodity and the price per unit of one.
 4. given the number of units of each, the combined price of both, and the ratio of the price of the one to that of the other.
 - d. To find the amount to be received for several items, given the price of each.
 - e. To make change, given an amount of money and the price of a commodity.
 - f. To find the margin or loss given the cost price and the selling price.
 - g. To find the total margin or total loss:
 1. given the number of units and the margin or loss per unit.
 2. given the unit cost, the unit selling price, and the number of units.
 - h. To find the margin or loss per unit, given the total margin or loss and the number of units.

²²Descriptions of quantitative relations given below are expressed in terms of buying. In some cases changes in terminology would be necessary if the activity were to be considered from the standpoint of selling.

²³"Total price" is used to designate the amount received for several units of the same commodity rather than the amount received for several commodities.

²⁴Price is used to designate the quantity taken as a basis of computation. Usually "price" refers to the value or worth of a unit rather than a specified number of units. "Price" is often limited by the qualifying terms cost, selling, marked, and list.

²⁵Margin is a term used to represent the difference between the cost price and the selling price and therefore is a substitute for the words "gain" and "profit" as they are commonly used.

B2 Buying and selling, more complex types.

- a. To find the selling price:
 1. given the rate²⁶ of discount or loss, and the price.
 2. given the rate of advance or margin and the price.
 3. given the rate of two or more successive discounts and the price.
 4. given the price, rate of advance or margin, and rate of discount or loss.
 5. given the rate of commission, discount, margin, or loss and the amount of commission, discount, margin, or loss.
 6. given the price and the amount of commission or discount.
- b. To find the amount of margin, loss, commission, or discount:
 1. given the total price and the rate of margin, loss, commission, or discount.
 2. given two or more successive discounts and the total price.
 3. given the total price and the selling price.
- c. To find the rate of margin, loss, discount, advance, or commission:
 1. given the total price and the amount of margin, loss, discount, advance, or commission.
 2. given the cost price in terms of two successive rates of discounts and the list price, and the selling price in terms of a single rate of discount and the list price.
 3. given the total price and the selling price.
- d. To find the price:
 1. given the selling price and the rate of discount or loss.
 2. given the amount of margin, loss, commission, or discount and the rate of margin, loss, commission, or discount.
 3. given the selling price and rate of margin.
 4. given the selling price and two or more successive discounts.
- e. To find the amount due the agent or agents, given the number of units, the price per unit, and the rate of commission.
- f. To find the equivalent single discount in percent, given two or more successive rates of discount.
- g. To find one of two or more successive discounts, given the list price, one or more of the successive discounts in percent, and the net price.

Limitations of the list of problem types. A comparison of the problem types appearing under the caption, operation problems, with those listed as activity problems reveals a number of apparent duplications. This is to be expected because it is theoretically possible for any question concerning a quantitative relationship listed under operation problems to be implied in a problem clearly identified with some activity. When this occurs the problem has been classified as belonging to a type under activity problems.

The recognition of two overlapping groups of problems appeared to be justified by the fact that many problems found in arithmetic texts could not be assigned to an activity and that when they were clearly identified with a particular activity such as "borrowing, lending or saving money" or "insurance" a technical terminology was introduced which tended to make them different from other problems requiring the same calculations but not identified with the same activity.

²⁶Rate may be expressed in terms of percent or as a fraction.

Failure to group problem types under some such heading as "activity problems" would suggest that the problems of arithmetic were abstract. The absence of a list of "operation problems" would have made it impossible to classify many problems now found in arithmetic texts.

A comparison of certain of the groups of problem types under activity problems (e.g., B1, B2 and B3) will reveal a type of duplication caused by the fact that two bases of differentiation were recognized; *first*, the general character of the activity in which problems occur, and *second*, the question concerning functional relations which a problem implies. It was decided that the first basis (general character of the activity) should have precedence over the second.

The writer and his assistants were compelled to exercise judgment on a number of other points. Consequently, the list of problem types should not be accepted as final. Especially, the conclusion that there are exactly 333 problem types should not be drawn. This total would have been different if different decisions on a number of minor points had been made.

Value of the list of problem types. Although the list of problem types has been evolved after much careful thought and has been used as a basis in analyzing ten series of arithmetic texts, the enumeration of problem types given in Appendix A must be considered only a tentative formulation representing the judgment of the writer and his assistants. However, this tentative formulation should prove useful because it emphasizes that an arithmetical problem asks a question concerning a quantitative relationship which the solver of the problem must identify and then answer. Furthermore, it provides a working basis for considering the problem content of arithmetics.

Conclusions in regard to learning exercises. The most significant conclusion to be drawn from this description of the learning exercises of arithmetic, especially the types of examples and problems, is that the number of types of exercises is large. Each type of exercise constitutes a basis for a learning activity which is different, at least in some respects, from that occurring in a pupil's response to any other type. Hence, this analysis of the learning exercises of arithmetic is a necessary prerequisite for a consideration of the teacher's responsibility for devising and selecting exercises.

CHAPTER IV

THE LEARNING EXERCISES PROVIDED BY TEXTS IN ARITHMETIC

Three types of content in arithmetic texts. Arithmetic texts include three general types of content; (1) statements of what pupils are to learn (facts, rules, definitions, and principles); (2) illustrations, explanations, descriptions, and the like which imply learning exercises involving tracing (see numbers 8, 20, and 26, page 34); and (3) explicit learning exercises. Examples (explicit requests to perform specified calculations) and verbal problems make up the majority of this third type of material.

The problem of this chapter. The problem of this chapter is to present certain information relative to the example and problem content of arithmetic texts. The information relating to provisions for the first type of learning exercise is taken from studies reported by other investigators. The information concerning the problem content is based on an original investigation conducted under the direction of the writer.

The example content of arithmetic texts. Since the principal function of examples is to provide practice on the combinations (basic and secondary), the most significant information relative to the example content of arithmetic texts is the number of occurrences of each of the combinations. A statement of the amount of space devoted to examples or the number of learning exercises of this type does not constitute a very significant description.¹ An analysis of the examples with respect to the operations involved is a little more helpful but it still leaves one with only a very vague notion of the nature of the learning exercises which the texts provide.

Writers² who have analyzed the example content³ of arithmetic

¹For an illustration of an analysis of this type see:

SPAULDING, F. T. "An analysis of the content of six third-grade arithmetics," *Journal of Educational Research*, 4:413-23, December, 1921. The investigator presents a count of the examples and problems in six third-grade arithmetics. He found that the ratio of examples to problems varied from nearly 5 to 1 to approximately 2 to 1, the average being a little more than 3 to 1.

²CLAPP, FRANK L. "The number combinations: their relative difficulty and the frequency of their appearance in textbooks." *Bureau of Educational Research Bulletin No. 2*. Madison, Wisconsin: University of Wisconsin, July, 1924. 126 p.

KNIGHT, F. B. "A note on the organization of drill work," *Journal of Educational Psychology*, 16:108-17, February, 1925.

texts agree that the provisions for practice on the different combinations vary greatly, and that pupils who do all of the examples provided by a given text will receive more practice on the easier combinations than on the more difficult ones.⁴ For example, Clapp⁵ reports the following frequencies of combinations in Book II of a certain series of arithmetics: $1 + 1 =$, 434 times; $2 + 1 =$, 444 times; $1 + 2 =$, 299 times; $4 + 1 =$, 447 times; $7 + 5 =$, 76 times; $7 + 6 =$, 74 times; $8 + 7 =$, 102 times; $6 + 8 =$, 62 times; $7 + 9 =$, 74 times. Osburn reports that "one hundred and eighty out of a total of 1,325 combinations do not occur at all in the book considered⁶ while some easy ones occur more than 300 times." Clapp reports the following coefficients of correlation between the difficulty of combinations and the frequency of their appearance in textbooks: addition $-.452 \pm .054$; subtraction $-.329 \pm .061$; multiplication $-.384 \pm .057$; division $-.421 \pm .061$. These results are for Text A. Similar coefficients of correlation are given for Text B. Since all of the coefficients are negative and "large" in comparison with the probable error, they mean the more difficult the combination the less frequently it occurs.⁷

As might be expected, when texts are compared with reference to their provisions for practice on the combinations, there is a conspicuous lack of similarity in their example content. With the possible exception of texts published since the results of the first analyses have been available, it appears certain that the practice a pupil receives upon the combinations of arithmetic, both basic and secondary, will not be adjusted to the difficulty of the combinations, and that the amount of practice upon the different combinations will depend upon the text he studies.

THORNDIKE, EDWARD L. *The Psychology of Arithmetic*. New York: The Macmillan Company, 1922, Chapter VI.

⁴"Example content" includes both examples as defined on page 19 and requests for the fundamental combinations.

⁵Since "easy" combinations are those which pupils respond to with the fewest errors and the "difficult" combinations are those which pupils know least well, one might insist that the combinations found to be "easy" possessed this quality because the texts provided much drill on them and that the "difficult" ones were not known so well because the pupils were not given as much opportunity to learn them. However, a careful study of the available data does not support this hypothesis. It appears that certain combinations are inherently more difficult than others.

⁶CLAPP, FRANK L. *op. cit.*

⁷This is described as Book I of a widely used series of arithmetics.

⁸An analysis of the practice exercises prepared by Curtis and by Studebaker reveals similar conditions. See:

OSBURN, W. J. "A study of the validity of the Curtis and Studebaker Practice Tests in the Fundamentals of Arithmetic," *Journal of Educational Research*, 8:93-105, September, 1923.

The distribution of practice. In considering the learning exercises provided by a series of arithmetics, it is important to note the distribution of practice as well as the nature of this practice. Thorndike⁸ has shown that, in certain texts which are probably representative, the practice is distributed in a way that appears to represent inefficient instruction. Investigations in the psychology of learning indicate that in learning the combinations of arithmetic there should be a reasonably large number of repetitions during the first learning period and a gradual decrease in their number during subsequent periods which should occur at gradually increasing intervals. Thorndike found that the amount of practice on 5×5 in the first two books of a three-book series increased as the pupil advanced through the series. He suggests that the distribution of practice in this combination "would be better if the pupil began at the end and went backwards."

Problem content of arithmetic texts. In order to determine the nature of the problem content⁹ of arithmetic texts, the list of 333 problem types described in Chapter III was used as a basis for analyzing the second and third books of ten three-book series of arithmetics.¹⁰ Each problem in these books was read and a decision made in regard to the question it implicitly asked concerning a functional relationship.¹¹

⁸THORNDIKE, EDWARD L. *The Psychology of Arithmetic*. New York: The Macmillan Company, 1922, Chapter VIII.

⁹"Problem content" does not include explicit requests for a definite calculation, such as "What is 7 percent of \$7400.00?" or "Reduce 2 miles to yards." Such exercises were considered examples.

¹⁰The series analyzed are:

ANDERSON, ROBERT F. *The Anderson Arithmetic*. New York: Silver, Burdett and Company, 1924.

ALEXANDER, GEORGIA, and DEWEY, JOHN. *The Alexander-Dewey Arithmetic*. New York: Longmans, Green and Company, 1921.

DRUSHEL, J. ANDREW, NOONAN, MARGARET E., and WITHERS, JOHN W. *Arithmetical Essentials*. New York: Lyons and Carnahan, 1921.

HAMILTON, SAMUEL. *Hamilton's Essentials of Arithmetic*. Higher Grades. New York: American Book Company, 1920.

HOYT, FRANKLIN S., and PEET, HARRIET E. *Everyday Arithmetic*. New York: Houghton Mifflin Company, 1920.

LENNES, N. G., and JENKINS, FRANCES. *Applied Arithmetic*. The Three Essentials. Philadelphia: J. B. Lippincott Company, 1920.

STONE, JOHN C., and MILLIS, JAMES F. *New Stone-Millis Arithmetic*. New York: Benjamin H. Sanborn and Company, 1920.

THORNDIKE, EDWARD LEE. *The Thorndike Arithmetics*. New York: Rand, McNally and Company, 1917.

WATSON, BRUCE M., and WHITE, CHARLES E. *Modern Arithmetic for Upper Grades*. New York: D. C. Heath and Company, 1918.

WENTWORTH, GEORGE, and SMITH, DAVID EUGENE. *Essentials of Arithmetic*. New York: Ginn and Company, 1915.

¹¹This work was done by Ollie Asher during the year 1924-25 under the immediate supervision of John A. Clark, Assistant in the Bureau of Educational Research.

TABLE II. NUMBER OF PROBLEMS IN THE TEXTS EXAMINED

Text	Number of Problems		
	Book II	Book III	Total
A.....	899	876	1775
B.....	1441	1482	2923
C.....	824	678	1502
D.....	1339	1953	3292
E.....	1134	1246	2380
F.....	1052	1008	2060
G.....	1269	1336	2605
H.....	1240	1184	2424
I.....	1186	1417	2603
J.....	1070	1363	2433
Total.....	11454	12543	23997

It was then classified under the problem type described by that question. (See page 41.)

Some of the problems asked relatively simple questions but in other cases the question was complex in the sense that its answer involved the specification of an extended series of calculations. Analysis of such "complex" problems revealed that in most cases they might be considered as consisting of a sequence of two or more simpler problems. Since it soon became apparent that unless some such policy were adopted, the number of problem types would be increased indefinitely, the more "complex" problems, amounting to slightly more than one-fourth of the total numbers, were classified as consisting of a sequence of two or more simpler problems. This procedure is illustrated by the following problems whose classification is given in the left-hand margin.¹²

B11a1(a) It cost Robert \$4.25 to grow the corn. He figures that it cost
A1 him 9 hours labor in selling the corn. Counting the labor of selling the corn at 8 cents an hour, what was the total cost of growing and selling the 234 ears?

A6a2 The children of the Mullanphy School collected in two months
B1a1₂ 11325 lb. of old newspapers and 2550 lb. of magazines. They received
A1 \$1.25 per 100 lb. for the newspapers and \$2.75 per 100 lb. for the magazines. What was the total received for old paper?

B11a1(a) Andrew's father worked for a farmer. He received \$50 a month for
A3 the 12 months of the year, free house rent worth \$23 a month, 12 bu.
B1a1₂ of potatoes worth \$1.30 a bushel, and 365 qt. of milk worth 8c a
A1 quart. What he received was equivalent to what money wages for the year?

¹²For a description of the problem types indicated by the symbols used, see Appendix A.

TABLE III. FREQUENCY OF OCCURRENCE OF PROBLEM TYPES

	Book II	Book III	Total
Total frequency of problem types occurring in simple problems.....	9107	8445	17552
Total frequency of problem types occurring in "complex" problems.....	11801	18655	30456
Total.....	20908	27100	48008

- B1c1 What percent of the cost does a newsboy make on papers that he
- B1f buys at the rate of 3 for 4c and sells at 2c each? What percent of
- B2c2 the selling price does he make? What percent of the selling price does
- A15₂ the news dealer receive? What percent of the selling price does the newsboy receive?

The total number of problems in each book analyzed is shown in Table II. According to Table III, 17,552 of the 23,997 problems were classified under some one of the 333 problem types. The remaining problems were considered "complex" and were classified as representing a combination of two or more problem types. The fact that 6,442 problems represent a total frequency of 30,456 problem types indicates that most of them were very "complex."

Results of the analysis of problem content. A detailed summary of the results of analyzing the ten series of arithmetic texts is given in Appendix A in the following form.

- A1 To find totals by addition, given two or more items, values, etc.
 - A82 B112 C62 D66 E132 F43 G49 H82 I46 J75 749
 - A416 B495 C489 D440 E532 F375 G290 H537 I387 J419 4380
- B1e To make change, given an amount of money and the price of a commodity.
 - A26 B23 C68 D15 E13 I1 J1 147
 - A28 B36 C96 D33 E36 G3 H3 I11 J12 258

The first line of the frequencies gives the number of occurrences of the problem type when not combined with another type, that is, in "simple" problems. The second line gives the total occurrences of the problem type in both "simple" and "complex" problems. The number of occurrences in "complex" problems may be found by subtracting the upper number from the lower. Each of the letters, A, B, C, D, E, F, G, H, I, J, indicates a particular text.

The detailed summary given in Appendix A should be studied in order to secure a clear idea of the nature of the problem content of the arithmetics analyzed. Only sixty out of 333 problem types appear in all of the ten series of arithmetics, and only twenty-five, ten or more times in all texts. A number appear in only one or two of the texts.

TABLE IV. FREQUENCIES OF PROBLEM TYPES AND NUMBER OF PROBLEM TYPES IN EACH TEXT

Text	Operation Problems		Activity Problems		Total	
	Total frequency of problem types	Number of problem types	Total frequency of problem types	Number of problem types	Total frequency of problem types	Number of problem types
A.....	2602	44	1900	129	4502	173
B.....	3298	35	1998	133	5296	168
C.....	2479	39	1564	122	4043	161
D.....	3046	38	2912	140	5958	178
E.....	2449	38	1842	179	4291	217
F.....	2046	35	1805	95	3851	130
G.....	2782	40	2042	137	4824	177
H.....	3793	37	1969	110	5762	147
I.....	2661	44	2059	112	4720	156
J.....	2613	37	2148	125	4761	162
Total.....	27769	387	20239	1282	48008	1669

Twelve types¹³ have total frequencies over 1000. The sum of the twelve frequencies is 29,964 or slightly more than three-fifths of the sum of all frequencies.

Table IV gives the number of problem types in each text and the sum of the frequencies. In interpreting this table, the reader should bear in mind that there are 52 problem types under operation problems and 281 under activity problems. The fact that all problem types do not appear in all texts is apparent from Appendix A. Table IV shows that two texts, A and I, include 44 of the 52 problem types under operation problems and that 35 in text F represents the lowest number of problem types. A somewhat more analytical summary of the operation problem content of the several texts is given by Table IV A. It is clear that the texts differ in respect to the problem types included and also in the frequency of the occurrence of the types. For example, A14 appears in all of the texts but in text B its frequency is 1 and in text H, 50.

The variability among the ten series of arithmetics is even greater in the case of the activity problems. Text F includes only 95 of the 281 problem types. The greatest number of problem types found in a single text is 179 in text E. The extent of the variability is more clearly indicated by Table IVB. It is obvious in all of the texts that there are

¹³These with their frequencies are: A1, 4380; A2, 4074; A3, 3532; A5, 1101; A6a, 1556; A7a, 1148; A10a, 1115; A15, 2172; A16, 2644; A19, 1364; B1a1, 4472; B4a, 2366.

TABLE IV. A. NUMBER OF PROBLEM TYPES IN EACH CLASS OF OPERATION PROBLEMS AND TOTAL FREQUENCY OF OCCURRENCE

Operations	Total Problem Types		A		B		C		D		E		F		G		H		I		J		Total	
	1	T*	F†	T	F	T	F	T	F	T	F	T	F	T	F	T	F	T	F	T	F	T	F	F
A1.....	1	1	416	1	489	1	440	1	532	1	375	1	290	1	537	1	387	1	419	1	4380			
A2.....	1	1	375	1	370	1	445	1	429	1	186	1	345	1	532	1	314	1	410	1	4074			
A3.....	1	1	415	1	230	1	314	1	307	1	305	1	399	1	560	1	334	1	315	1	3552			
A4.....	1	1	398	1	171	1	91	1	66	1	66	1	164	1	104	1	56	1	47	1	8859			
A5.....	1	1	391	1	172	1	57	1	64	1	70	1	187	1	154	1	84	1	144	1	1101			
A6.....	2	2	200	2	164	2	269	2	128	2	229	2	223	2	141	2	212	2	182	2	1878			
A7.....	2	2	178	2	150	2	188	2	132	2	100	2	130	2	116	2	167	2	177	2	1365			
A8.....	11	10	51	6	15	7	18	7	39	6	35	8	67	5	11	10	57	8	40	8	340			
A9.....	1	1	93	4	127	4	185	5	103	4	120	5	219	6	181	7	115	4	111	4	1346			
A10.....	7	6	16	5	20	5	35	5	32	5	20	5	49	3	10	4	49	3	15	3	298			
A11.....	4	3	16	5	32	4	44	5	36	5	81	5	156	4	77	5	127	5	55	5	793			
A12.....	7	6	51	2	30	1	111	5	36	5	81	5	156	4	77	5	127	5	55	5	793			
A13.....	1	1	8	1	11	1	11	1	8	1	5	1	5	1	5	1	8	1	39	1	145			
A14.....	1	1	199	1	145	1	327	1	193	1	178	1	162	1	48	1	151	1	39	1	2172			
A15.....	1	1	283	1	435	1	278	1	152	1	110	1	196	1	51	1	315	1	132	1	2643			
A16.....	1	1	144	1	145	1	43	1	49	1	33	1	26	1	51	1	54	1	103	1	567			
A17.....	1	1	116	1	170	1	117	1	117	1	74	1	131	1	155	1	14	1	14	1	1361			
A18.....	1	1	114	1	124	1	145	1	95	1	74	1	131	1	155	1	165	1	152	1	1361			
A19.....	1	1	114	1	124	1	145	1	95	1	74	1	131	1	155	1	165	1	152	1	1361			
A20.....	1	1	114	1	124	1	145	1	95	1	74	1	131	1	155	1	165	1	152	1	1361			
A21.....	1	1	114	1	124	1	145	1	95	1	74	1	131	1	155	1	165	1	152	1	1361			
A22.....	1	1	114	1	124	1	145	1	95	1	74	1	131	1	155	1	165	1	152	1	1361			
A23.....	1	1	114	1	124	1	145	1	95	1	74	1	131	1	155	1	165	1	152	1	1361			
A24.....	1	1	114	1	124	1	145	1	95	1	74	1	131	1	155	1	165	1	152	1	1361			
A25.....	1	1	114	1	124	1	145	1	95	1	74	1	131	1	155	1	165	1	152	1	1361			
Total F.....	52	44	2602	35	3298	35	3046	38	2449	35	2046	40	2782	37	3793	44	2661	37	2613	44	27769			
Total T.....	52	44	2602	35	3298	35	3046	38	2449	35	2046	40	2782	37	3793	44	2661	37	2613	44	27769			

*Problem types appearing in the text.

†Total frequency.

TABLE IV B. NUMBER OF ALL PROBLEM TYPES IN EACH CLASS OF ACTIVITY PROBLEMS AND TOTAL FREQUENCY OF OCCURRENCE

Activities	Total Problem Types		A		B		C		D		E		F		G		H		I		J		Total
	T*	F†	T	F	T	F	T	F	T	F	T	F	T	F	T	F	T	F	T	F	T	F	F
B1.....	12	631	12	838	674	11	957	13	669	9	397	12	651	10	738	10	586	11	886	11	886	7027	
B2.....	14	402	14	373	174	12	455	13	139	10	266	14	360	8	313	13	183	14	275	14	275	2940	
B3.....	15	46	19	75	39	14	93	25	165	11	66	21	93	8	30	8	59	14	30	14	30	696	
B4.....	11	271	11	212	168	12	574	13	269	11	597	7	325	7	289	9	529	12	359	12	359	3593	
B5.....	22	87	8	94	105	15	192	15	112	11	133	16	127	12	37	14	211	10	53	10	53	1151	
B6.....	18	7	98	118	41	10	201	13	114	8	73	10	197	9	93	11	96	11	140	11	140	1171	
B7.....	23	8	85	9	75	14	86	13	96	7	79	6	34	6	66	11	88	7	62	7	62	773	
B8.....	8	1	2	15	18	1	1	6	14	3	17	3	10	2	3	2	2	2	2	2	2	57	
B9.....	6	3	4	15	8	3	37	5	10	2	14	3	8	3	16	4	72	4	30	4	30	236	
B10.....	35	18	81	24	38	15	108	20	68	10	40	18	82	12	111	12	53	17	141	17	141	746	
B11.....	6	65	17	55	64	9	63	9	86	3	24	4	40	7	130	6	66	5	59	5	59	652	
B12.....	4	64	4	87	47	4	105	3	26	3	89	3	50	3	1	29	3	69	3	69	3	79	645
B13.....	62	18	42	55	113	20	40	31	74	7	10	22	65	25	114	9	45	17	34	17	34	592	
Total F.....				1998	1564	140	2912	179	1842	95	1805	137	2042	110	1969	112	2059	125	2148	125	2148	20239	
Total T.....	281	1900	133	122	1564	140	2912	179	1842	95	1805	137	2042	110	1969	112	2059	125	2148	125	2148	20239	

*Problem types appearing in the text.

†Total frequency.

“gaps” in the problem content of each of the texts analyzed, as well as variations in the frequency of occurrence of the problem types included.

The vocabulary of problems belonging to a type. The reader should not assume that the problems classified under a given type approach identity. There are many differences in vocabulary and other phases of verbal expression. The problems assigned to B1a1¹⁴ were examined and after language duplicates were eliminated, 249 different problems remained. This examination revealed a large number of ways of stating the question asked. Some of the more frequently used forms are given below:¹⁵

- Find the cost of (38)
 - the value of (5)
 - the amount (3)
- How much did...cost (32)
 - much must...pay for (19)
 - much did...receive for (12)
 - much is...worth (7)
 - much did...get for them (4)
 - much should be charged for (3)
- What is the cost of (30)
 - does...pay for (12)
 - is the value of (5)
 - is the...bill for (4)
 - is the exact cost (4)
 - was received for them (3)
 - was...worth (3)

A vocabulary analysis of the 249 illustrative problems showed that 612 different words and phrases were used. Over one hundred of these words and phrases were used with a technical meaning. This list included words such as:

all..... (8)	find.....(59)	receive.....(15)
amount.....(12)	following.....(12)	received..... (7)
apiece.....(14)	how much.....(98)	selling..... (9)
average.....(11)	paid.....(12)	sold.....(33)
bought.....(19)	pay.....(29)	use..... (5)
buys.....(12)	per.....(41)	used.....(13)
cost.....(149)	price.....(17)	value.....(18)
each.....(40)	purchases..... (7)	weighing..... (7)
exact..... (7)	rate..... (8)	

¹⁴“To find the total price, given the number of units and price per unit.” The total number of “simple” problems classified under this type was 1195.

¹⁵The numbers in parentheses represent the frequency of occurrence in the 249 illustrative problems.

Conclusions concerning problem content of arithmetic texts. The data relative to the problem content of arithmetic texts presented in the preceding pages appear to justify the following conclusions.

1. The "average" three-book series of arithmetics includes about 2400 verbal problems in the second and third books. This provides an average of 600 learning exercises per year.

In the ten series analyzed, the number of verbal problems varied from 1775 to 3292. In six of the series the number of problems in the third book was greater than in the second.

2. Although 333 problem types were recognized, the average number per series was only 167. The lowest number included in any series was 130; the highest, 217. Only 60 problem types were found in all ten series, 101 were common to eight or more of the series, and 12 furnished over three-fifths of the sum of frequencies. It is, therefore, obvious that the verbal problems of our arithmetics afford pupils extensive opportunity to become acquainted with a few problem types, (probably not more than 20 to 25)¹⁶ but beyond this limited number, the opportunities for becoming acquainted with functional relationships vary greatly with the text. If the problems of a series of arithmetics are compared with the list of problem types given in Appendix A, from one to two hundred of these types will not be found.

¹⁶There are only 25 problem types which have a frequency of 10 or more for each text.



CHAPTER V

THE TEACHER'S RESPONSIBILITY FOR DEVISING AND SELECTING LEARNING EXERCISES IN ARITHMETIC

General statement of a teacher's responsibility. When the immediate objectives to be attained by her pupils have been established and a textbook adopted, the teacher becomes responsible for devising or discovering in other sources the learning exercises needed in addition to those provided by the text. She is also responsible for selecting from the available learning exercises those which will provide the most efficient basis for the attainment of the objectives. This second responsibility is especially important as a means of adapting instruction to individual differences.

The magnitude of the teacher's responsibility becomes apparent when we recall that, although arithmetic texts provide a large number of examples and verbal problems, there are many "gaps" in these types of learning exercises and that a complete analysis of the textbooks would doubtless reveal an inadequate supply of many if not all of the types of learning exercises listed on pages 33-35. It may occur to the reader that a "perfect" text would absolve the teacher of all responsibility in connection with devising and selecting learning exercises but it does not appear that a millenium can be attained when the same text is prescribed for all members of a class. Even in a so-called homogeneous group, the needs of all pupils for learning exercises will not be identical. Their needs will differ with respect to both number and kind of exercises. However, the more "perfect" the provisions of the text and the more homogeneous the class, the less the teacher's responsibility for devising and selecting learning exercises.

Before attempting a more detailed statement of the teacher's responsibility for devising and selecting learning exercises, it will be helpful to note certain general principles.

1. **Factors affecting the kind and number of learning exercises needed.** It is of course obvious that in engendering a given ability or group of abilities the exercises assigned should be of such a character that the resulting learning activity will produce the desired outcomes. Hence the immediate objectives constitute the first factor to be considered in deciding the kind of learning exercises needed. If the pupil is expected to acquire the ability to do long column addition without

carrying, it is generally agreed he should engage in the activity of adding single columns of figures of approximately the length specified by the objective and that these examples should include all of the "higher decade" combinations that are included in this objective.

The number of examples which a given pupil should do depends upon the degree of facility to be acquired. If the pupil is to acquire a degree of skill equivalent to that possessed by an expert bookkeeper, a relatively large number of learning exercises will be required. On the other hand, if a lower standard is set, for example, the degree of skill possessed by the average literate adult, a much smaller number of examples will suffice.

The dependence of the kind and number of learning exercises upon the objectives is not so apparent in all cases as that noted in the preceding paragraph, but the acceptance of the principle that learning is an active process implies the corollary that a connection between a stimulus or situation and a response is established only by exercising it. Hence, what a child learns is not the product of learning activity in general but of the particular activities in which he engages and therefore the learning exercises assigned should be devised and selected in accord with the recognized objectives of the subject.

In Chapter I, the specific character of arithmetical objectives was emphasized. For example, it was pointed out that the responses to $7 + 5 =$, $5 + 7 =$, $15 + 7 =$, and $25 + 7 =$ involve the functioning of different abilities. However, such abilities are undoubtedly related. The pupil who is able to respond fluently to $5 + 7 =$ will more easily learn to respond to $15 + 7 =$, $25 + 7 =$ and the like than if he did not know $5 + 7 = 12$. Similarly the learning of addition combinations facilitates the learning of the corresponding subtraction combinations. Hence, the number of learning exercises necessary to engender a specified ability depends upon the amount of facilitation afforded by abilities previously acquired.

The degree of satisfaction a pupil experiences in doing the exercises assigned also affects the number needed¹ and hence, indirectly, the kind that should be assigned. If the exercises and the conditions under which they are done are such that the pupil derives satisfaction from doing them, fewer exercises will be needed than if the pupil is indifferent or bored by the activity.

Our information concerning individual differences makes it clear that some children learn more rapidly than others. This means that the exercise of a connection between a stimulus and a response produces

¹See Laws of Learning, pages 31-32, for the basis of this statement.

a greater effect in some pupils than in others, and hence the number of repetitions required to "establish" a given connection varies with the capacity to learn arithmetic. This is especially true in the acquiring of general and abstract meanings. The kind of learning exercises needed also varies with the capacity to learn arithmetic. Gifted children are quick to grasp abstract meanings and general relations and hence do not need to do some of the exercises that are essential for pupils of lesser capacity. Furthermore, gifted children exhibit considerable initiative and resourcefulness in devising things to do without explicit assignment. On the other hand, pupils of less than average capacity to learn and frequently those of average capacity may require detailed explanations or demonstrations. Sometimes it will be necessary to provide opportunities for perceptual experiencing.

A pupil's need for learning exercises at any given period in his educational career depends upon his previous experience both in and out of school. For example, it has been found in teaching spelling that a considerable percent of the words specified by the course of study for a given grade can be spelled correctly before they are assigned for study. Investigations would doubtless reveal a somewhat similar condition in the field of arithmetic, especially with reference to general and abstract meanings. A pupil who has lived on a farm and become familiar with terms such as field, yield per acre, bushel, pound, mile, harvest, and so forth has needs for perceptual experiences very different from those of a boy who has never seen a farm. A son of a grocer who has become familiar with his father's store, perhaps by working in it, learns little from dramatizing a grocery store in the school.

~ **Neither capacity to learn arithmetic nor previous experience definitely known.** Although we have a number of instruments for measuring general intelligence, evidence indicates that capacity to learn arithmetic does not correlate perfectly with the capacity for learning in general.² This condition, added to the fact that intelligence test scores involve errors (sometimes relatively large) when considered as measures of general capacity to learn, makes it clear that these instruments will not yield very accurate measures of capacity to learn arithmetic. It is possible that more satisfactory information might be secured by means of a test designed to measure capacity to learn arithmetic but such instruments have not been constructed.

²DEV OSS, JAMES C. "Specialization of the abilities of gifted children." *Genetic Studies of Genii*. Vol. I. Stanford University, California: Stanford University Press, 1925, Chapter XII.

It is likewise impossible to obtain a detailed and accurate record of a pupil's previous experience. Previous school records, particularly the grades in arithmetic, are indicative of school experience. Some information concerning experience outside of school can be obtained by questioning the pupil and by ascertaining the possibilities of his environment. However, the information obtained will seldom if ever approach a complete description of a pupil's previous experiences that have contributed arithmetical controls of conduct.

Detailed determination of a teacher's responsibility for devising and selecting learning exercises a highly complex task. Recognition of the factors affecting pupil needs makes it clear that the task of determining in detail the teacher's responsibility for devising and selecting learning exercises for a class in arithmetic is highly complex. Furthermore, the varying provisions of arithmetic texts make impossible a detailed statement that would be applicable to all situations. However, a teacher must make a decision in regard to the exercises to be assigned to the various members of her class. To assist her in this task certain general statements may be made.

4. The attainment of specific habit objectives requires repetition. When the objective to be attained is the formation of a specific habit, the learning exercises must provide for a considerable number of repetitions or practices of each of the bonds connecting a response with a situation or stimulus, such as $4 \times 8 = 32$. The significance of this statement becomes more apparent when the range of specific habit objectives is recalled. (See Chapter I.) Other things being equal the strength of the connection increases with the number of repetitions but is also affected by other factors. Hence, in general the number of repetitions necessary to attain the different objectives will vary.

Thorndike's estimate of learning exercises for certain objectives. Thorndike³ has estimated that in the case of an average pupil 67 repetitions of an easy connection⁴ such as " $2 \times 5 = 10$, or $10 - 2 = 8$, or the double bond $7 = \text{two } 3\text{'s and } 1 \text{ remainder,}$ " will be sufficient to attain, by the end of the sixth grade, the objective of 99.5 percent accuracy in the functioning of the connection, provided the repetitions are properly distributed and "the teaching is by an intelligent person working in accord with psychological principles as to both ability and interest." The distribution of the repetitions recommended is as follows:

³THORNDIKE, E. L. *The Psychology of Arithmetic*. New York: The Macmillan Company, 1922, p. 133.

⁴The reader should note that the connections or bonds referred to are facilitated by other bonds to a relatively high degree. This is one reason why they are "easy."

twelve during the first week, twenty-five during the two months following and thirty spread over the remaining period. For the more difficult combinations⁵ 100 repetitions are estimated as necessary. Gifted children will require fewer repetitions and those who possess less than average capacity to learn, a larger number of repetitions.

4* **Isolated practice versus application.** In considering the amount of practice needed, it is important to bear in mind that repetitions occur in the application of a habit as well as when it is singled out for isolated practice. For example, the addition combinations are exercises in doing addition examples involving several figures to the column as well as when the pupil responds to $6 + 7 = ?$, $9 + 8 = ?$, $13 + 5 = ?$, $24 + 7 = ?$. and the like. The combinations are exercised also in performing the calculations required in solving verbal problems. Hence, the total number of repetitions of a particular combination is the number of repetitions in isolation plus the frequency of its occurrence in examples and in the calculations required by verbal problems.

Learning exercises needed for attainment of problem solving objectives. In solving verbal problems, it is necessary that one comprehend the statement of the problem. This requires that he must know the meaning of the technical words and phrases used, especially those which describe quantitative relations. The problem solver must detect and comprehend the question or questions concerning functional relationships which are implied in the statement of the problem and then provide the answer which will be the specification of the calculations to be performed. It, therefore, appears that the problem-solving objectives include knowledge of the technical vocabulary used in stating problems plus the ability to detect, formulate and answer questions concerning a number of functional relationships.⁶

For responding to practical situations, a somewhat different combination of abilities is required. A reading vocabulary is not necessary but one who is efficient in dealing with practical situations probably uses words as symbols of ideas and other knowledge elements in his thinking. Hence, mastery of a technical vocabulary occupies a place

⁵There have been several investigations to determine the relative difficulty of the combinations. The most comprehensive study is by:

CLAPP, FRANK L. "The number combinations: their relative difficulty and the frequency of their appearance in textbooks." Bureau of Educational Research Bulletin No. 2. Madison, Wisconsin: University of Wisconsin, July, 1924. 126 p. In general, the more difficult combinations are those involving the large numbers but there are many exceptions especially those involving division combinations like: $2 \div 2 =$, $8 \div 8 =$, and $2 \div 1 =$.

⁶See Appendix B for suggested minimum essential list of functional relationships.

in the abilities required for responding to practical situations involving arithmetical problems but probably a less extensive vocabulary suffices since the problem solver is not required to respond to verbal statements formulated by another person. The need for answering questions concerning functional relationships appears to be the same for both verbal problems and practical situations, provided the answer to the problem is to be obtained by calculation and not by using some mechanical device or special rule.

Although ability to solve problems may include other elements, knowledge of technical vocabulary and ability to detect, formulate, and answer questions concerning functional relationships provides a basis for certain observations concerning the learning exercises needed for the attainment of the problem-solving objectives. The outstanding type of learning exercise now in use is the verbal problem. Current practice appears to assume that the abilities required for solving problems can be acquired efficiently by responding to verbal problems, and the predominant test of the response is the correctness of the numerical answer. Sometimes the pupil is asked to explain, which means that he is to describe or give a record of his thinking. A somewhat different type of learning exercise is set for the pupil when he is asked to read or listen to an explanation of a problem. A few other types of learning exercises⁷ are used but usually only to a very limited extent.

The nature of the objectives of problem solving suggests the need for exercises designed to provide an explicit basis for learning the technical terms of arithmetic. It is true that pupils do acquire the meanings for a number of technical terms by solving verbal problems but it appears that their achievements in this field are very inadequate. Recognition of this condition is evidenced by the common assertion that the reason why pupils are lacking in ability to solve problems is their inability to comprehend the statements of the problems. Vocabulary tests have corroborated this belief.⁸ It, therefore, appears that the reading of problems should be made an explicit learning activity. In testing this ability to read, pupils may be asked to restate the problem, substituting other technical terms when this is possible; to enumerate the quantities mentioned in the problem, specifying which ones are given and which one is to be found; to formulate the question (usually implied) concerning functional relationships.

⁷These include "problems without numbers," questions concerning functional relationships, projects, collecting problems from newspapers and other sources, and the like.

⁸CHASE, SARA E. "Waste in arithmetic," *Teachers College Record*, 18:360-70. September, 1917.

The description of the process of solving an arithmetical problem as "detecting and formulating the question concerning a functional relationship, plus answering this question," suggests that the more important functional relationships should be explicitly taught as principles. This will probably require the use of such questions as: "What calculations must be performed to find the area of a rectangle when the base and altitude (or two adjacent sides) are given?" "How does one find the rate of profit, given the cost of goods, and the expenses and losses?" "What calculation must be made to find the time required to travel a specified distance, given the distance and rate of travel?"

✧ **What problems should be used as learning exercises.** It is generally agreed that problems differ in their merits as learning exercises. Formerly authors of arithmetic texts included many problems that did not occur in the activities of adult life. Some of these were verbal puzzles designed to "sharpen the wit" of the pupils; others were derived from obsolete activities; and a third group, although identified with an adult activity, asked questions that would never arise because the answer would always be known in the practical situation. These kinds of problems appear to be less effective as learning exercises than real problems, that is, problems similar to those that arise in the normal activities of children and adults. ✓ "Similar" means not only that the problem involves the same functional relations but also that the quantities (prices, amounts, and so forth) be in substantial agreement with those of real life.

Problems may be real and still differ in ways that affect their value as learning exercises. The analysis of ten texts described in the preceding chapter showed the variety of vocabulary used and the degree of complexity of many of the problems appearing in current texts. The desirability of an extensive technical vocabulary is largely a matter of objectives. If the objectives of arithmetic specify one thousand technical terms to be mastered by the end of the eighth grade, these terms should be used in the stating of the verbal problems used as learning exercises. If a small number of technical terms is considered satisfactory, there should be less variety in the language of the verbal problems.

The objectives of arithmetic not yet determined in detail. The objectives of arithmetic were described at some length in Chapter I. Reference was made to investigations by Osburn and others (see page 9) which indicate that the number of combinations to be learned is much greater than commonly supposed. Certain functional relationships in problems were listed (see page 41) and the vocabulary used

in stating problems partially analyzed. It was observed that "ability to solve problems" means ability to solve a problem that has not been solved before. Doubtless the reader of the preceding pages probably has wished for a more complete and definite exposition of the objectives of arithmetic. With the exception of the skills that function in calculating, the description of objectives was in very general terms, especially in the case of those relating to problem solving. Even in the case of the calculation skills used with integers, the objectives have not been completely determined, and in the case of fractions and decimals less is known concerning what a pupil should learn.⁹ Hence, in attempting to determine the number and kind of learning exercises needed in the field of arithmetic, the assistance to be derived from formulations of objectives is limited. However, the exposition of objectives in Chapter I, supplemented by the analysis of examples and problems presented in Chapter IV, should lead the reader to formulate a concept of the objectives of arithmetic which will be very helpful in determining the number and kind of learning exercises needed.

An estimate of the teacher's responsibility for devising and selecting learning exercises in arithmetic. The kind and number of learning exercises needed in arithmetic depend upon several factors including the objectives, pupil's capacity to learn arithmetic, and his previous school experience. The preceding discussion should make it clear that our present state of ignorance about the various factors makes it impossible to do more than estimate the learning exercises needed. The estimate given here is in general terms and represents only the writer's judgment. Furthermore, the reader should bear in mind that the details of the teacher's responsibility for devising learning exercises will be affected by the text adopted for use.

Usually it will be necessary for the teacher of arithmetic to devise some exercises to provide perceptual experiences. In the primary grades opportunities for counting, measuring, and estimating should be provided. In the intermediate and upper grades the teacher may require pupils to observe adult activities that produce arithmetical problems and to give a description of these experiences but frequently it will be advisable to ask the pupils to listen to, or read, accounts of the experiences of others. An important phase of the devising of exer-

⁹The norms established for standardized tests constitute definite objectives but they are available for relatively few types of examples (see page 35). For a summary of types of examples for which norms are available see:

HERRIOTT, M. E. "How to make a course of study in arithmetic." University of Illinois Bulletin, Vol. 23, No. 6, Bureau of Educational Research Circular No. 37. Urbana: University of Illinois, 1925. 50 p.

cises of the latter type is the preparation or selection of the descriptions that the pupils are asked to listen to or read. The authors of some texts suggest construction exercises, games, and the dramatization of certain adult activities, but it is usually necessary for the teacher to plan at least the details of these types of exercises. In some cases the textbook will give little or no assistance. However, it may be noted that construction exercises, games, and the dramatization of adult activities may not be highly efficient learning exercises, especially after the primary grades are passed.

Arithmetic texts provide for practice in reading and copying numbers, supplying the missing number from specific quantitative relationships, and calculating, but analyses of the example content of texts show that there are "gaps" in the practice on the combinations, both basic and secondary. The teacher is responsible for discovering the "gaps" in the adopted text and for devising the exercises necessary to round out the practice. Usually the practice to be provided is on the more difficult basic combinations and certain of the secondary combinations. Several sets of practice exercises have been devised to relieve the teacher of a portion of this responsibility but investigation has revealed that some of the sets are not satisfactory. In some cases it will be necessary for the teacher to provide additional exercises in reading and especially in copying numbers.

The "gaps" in the verbal problems provided by a text vary but the analysis of the problem content reported in the preceding chapter indicates that the teacher will need to provide some additional problems. However, it appears likely that the number needed will not be large and pupils may be requested to collect some of the needed problems. On the other hand, a number of omissions usually will be justified. There is no justification for the very high frequency of certain problem types (see page 51) and some of the problem types listed in the Appendix are not sufficiently valuable to be included in our objectives.

The teacher will need to supply exercises that will lead to the mastery of functional relationships (see page 19), rules, definitions, and the like. Among these are requests to explain, explanations to be listened to, and thought questions. Other types of learning exercises for which the teacher must assume at least some responsibility are requests to check calculations, to inspect and verify solutions of problems, to collect quantitative information, and to generalize experience. Another very important responsibility of the teacher of arithmetic is to provide exercises that will lead to the mastery of the abstract and general terms in this field, especially those that are used in stating verbal problems.

APPENDIX A

Types of Functional Relations Found in the Problems of Ten Series of Arithmetics. The first line of numbers gives the frequencies of the simple occurrences of the relation, the second line the frequencies of the total occurrences of the relation. For a description of the method of analysis see page 48.

A. OPERATION PROBLEMS

- A1 To find totals by addition, given two or more items, values, etc.
- | | | | | | | | | | | |
|------|------|------|------|------|------|------|------|------|------|------|
| A82 | B112 | C62 | D66 | E132 | F43 | G49 | H82 | I46 | J75 | 749 |
| A416 | B495 | C489 | D440 | E532 | F375 | G290 | H537 | I387 | J419 | 4380 |
- A2 To find the difference, given two items, values, etc.
- | | | | | | | | | | | |
|------|------|------|------|------|------|------|------|------|------|------|
| A96 | B134 | C60 | D68 | E123 | F30 | G72 | H168 | I31 | J50 | 837 |
| A395 | B648 | C370 | D445 | E429 | F186 | G345 | H532 | I314 | J410 | 4074 |
- A3 To find the amount, or number needed, by multiplication, given a magnitude and the number of times it is to be taken.
- | | | | | | | | | | | |
|------|------|------|------|------|------|------|------|------|------|------|
| A53 | B96 | C20 | D36 | E75 | F47 | G107 | H161 | I45 | J38 | 678 |
| A415 | B353 | C230 | D314 | E307 | F305 | G399 | H560 | I334 | J315 | 3532 |
- A4 To find the size of a part of a magnitude, given the magnitude and the number of parts into which it is to be divided. (Averages which are the result of division only are included here.)
- | | | | | | | | | | | |
|-----|-----|------|-----|-----|-----|------|------|-----|-----|-----|
| A28 | B36 | C39 | D34 | E30 | F44 | G86 | H53 | I22 | J15 | 387 |
| A58 | B76 | C131 | D91 | E66 | F66 | G164 | H104 | I56 | J47 | 859 |
- A5 To find how many times a stated quantity is contained in a given magnitude, given the quantity and the magnitude.
- | | | | | | | | | | | |
|-----|------|-----|-----|-----|-----|------|------|-----|------|------|
| A16 | B59 | C11 | D15 | E26 | F8 | G37 | H82 | I29 | J26 | 309 |
| A91 | B148 | C72 | D57 | E94 | F70 | G187 | H154 | I84 | J144 | 1101 |
- A6 To find how many when reduction ascending is required, given
- a. a magnitude expressed in terms of a single denomination.
- | | | | | | | | | | | |
|------|------|------|------|------|------|------|------|------|------|------|
| A6 | B9 | C1 | D2 | E20 | F6 | G27 | H10 | I16 | J7 | 104 |
| A163 | B125 | C118 | D220 | E115 | F151 | G191 | H125 | I167 | J181 | 1556 |
- b. a magnitude expressed in terms of two or more denominations.
- | | | | | | | | | | | |
|-----|----|-----|-----|-----|-----|-----|-----|-----|----|-----|
| A11 | B3 | C1 | D5 | F4 | G2 | H5 | I13 | J1 | 45 | |
| A37 | B5 | C46 | D49 | E13 | F78 | G32 | H16 | I45 | J1 | 322 |
- A7 To find how many when reduction descending is required, given
- a. a magnitude expressed in terms of a single denomination.
- | | | | | | | | | | | |
|-----|------|-----|------|-----|-----|------|------|------|------|------|
| A6 | B20 | C2 | D17 | E16 | G14 | H5 | I27 | J13 | 110 | |
| A97 | B152 | C72 | D148 | E90 | F63 | G116 | H101 | I145 | J164 | 1148 |
- b. a magnitude expressed in terms of two or more denominations.
- | | | | | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| A6 | B4 | C2 | D3 | E8 | F5 | G6 | H4 | I10 | J2 | 50 |
| A31 | B11 | C22 | D40 | E12 | F37 | G14 | H15 | I22 | J13 | 217 |
- A8 To find a dimension, given
- a. the area of a rectangle and one side.
- | | | | | | | | | | | |
|-----|----|----|-----|-----|-----|-----|----|----|-----|-----|
| A10 | B3 | D2 | E14 | F6 | G5 | H3 | J7 | 50 | | |
| A10 | B6 | C2 | D8 | E26 | F13 | G15 | H3 | I4 | J15 | 102 |
- b. the area of a square.
- | | | | | | | | | |
|----|----|----|----|----|----|-----|----|----|
| B1 | E1 | G3 | I2 | J2 | 9 | | | |
| A2 | B1 | D1 | E2 | F2 | G6 | I14 | J4 | 32 |

c.	the area of a right triangle and one leg.										
	A1										1
d.	the two legs of a right triangle.										
	A2	B3	C6	E2	F8	G7	H1	I5	J1	35	
	A7	B4	C14	D1	E4	F9	G10	H1	I12	J6	68
e.	one leg and the hypotenuse of a right triangle.										
		B2	C2		F3	G2		I3	J1	13	
	A2	B2	C2			G4		I4	J1	18	
f.	the cubic contents of a parallelopiped and two dimensions.										
				E2	F2		H1	I7	J5	17	
	A4	B1	C5	D5	E2	F5	G13	H4	I10	J9	58
g.	the perimeter of any equilateral figure.										
		B1	C1	E1		G1		I1	J1	6	
	A1	B1	C2	D1	E1		G2		I1	J2	11
h.	the circumference of a circle.										
	A2		C1	E2	F3	G6		I1	J1	16	
	A2		C1	E3	F3	G8		I4	J2	23	
i.	the area of a circle.										
						G4		I1		5	
	A1					G9	H1	I2		13	
j.	the cubic contents of cylinder, and the diameter.										
	A1		C1	D1	E1			I1		5	
k.	the cubic contents and area of the base of a prism or cylinder.										
			D1					I4		5	
			D1				H2	I5	J1	9	
A9	To find a diameter equal to two or more smaller diameters, given the smaller diameters.										
							H1			1	
							H2			2	
A10	To find the area, given										
a.	dimensions of a square, rectangle or parallelogram.										
	A11	B10	C4	D15	E28	F3	G8	H47	I9	J14	149
	A73	B118	C72	D166	E85	F106	G161	H159	I73	J102	1115
b.	the base and altitude of a triangle. (Includes right triangle with two legs given.)										
	A1	B6	C1	D2	E2		G6	H8	I1		27
	A7	B7	C11	D15	E5	F1	G22	H11	I7	J2	88
c.	the diameter of a circle.										
				E1	F1				I2		4
	A7	B2	C5	D1	E9	F8	G20	H8	I16	J6	82
d.	the diameter of a sphere.										
			C1		F1				I3		5
	A3		C2		F4	G3	H2		I6		20
e.	the perimeter of a cone or pyramid, and its slant height.										
			C1	D1		G1			I2		2
	A2								I5		10
f.	the altitude and two bases of a trapezoid.										
				E1					I1		2
			C1	D2	E4	F1	G12	H1	I7	J1	29
g.	the altitude of a cylinder and radius of the base.										
	A1										1
	A1									I1	2

A11 To find the perimeter, given

a. one side of any equilateral figure.

A4	B4	C1	D1	E5	F2	G1		I7	J3	12
	B6	C3	D5	E7		G4				41

b. two adjacent sides of a rectangle or parallelogram.

A5	B11	C6	D10	E3		G5	H4	I8	J1	53
A11	B21	C11	D26	E25		G13	H6	I24	J7	144

c. the diameter or radius of a circle.

	B6	C1	D1	E1	F8	G7		I6		30
A1	B15	C6	D4	E9	F18	G32	H1	I17	J5	108

d. three sides of a triangle.

			E1					I1		2
			E1				H3	I1		5

A12 To find the cubic contents, given

a. one dimension of a cube.

A1			D3							4
----	--	--	----	--	--	--	--	--	--	---

b. 1. the three dimensions of a rectangular solid, such as room, bin, wood-pile, etc.

A10	B3	C8	D15	E6	F5	G9	H37	I18	J11	122
A37	B25	C49	D84	E29	F54	G80	H65	I96	J48	567

2. the area of one surface of a rectangular solid and the depth or altitude.

A1		C2	D1	E2	F1		H1	I7		15
----	--	----	----	----	----	--	----	----	--	----

c. the altitude and diameter or radius of a cylinder.

A3	B1	C3	D4	E1		G4	H4	I4	J1	25
A10	B5	C17	D22	E3	F18	G45	H9	I18	J4	151

d. the diameter or radius of a sphere.

					F4	G8	H2	I1		3
A1							H2	I5		20

e. the area of the base of a prism or cylinder and its altitude.

						G1				1
			D1	E1		G11			J3	16

f. the area of the base of a pyramid or cone, and its altitude.

				E1						1
A1		C1		E1	F4	G12		I1		20

A13 To find a difference, given denominate numbers of different denominations.

	C1									1
	C1									1

A14 To find the average given a series of items. (To find the average, given the total amount and the number of items is classified as A4.)

A7	B1	C7	D3	E6	F3	G5	H31	I3	J13	79
A8	B1	C14	D5	E8	F5	G5	H50	I8	J39	143

A15 To find the ratio of one number to another, given the two numbers.

A73	B52	C41	D116	E125	F80	G66	H264	I53	J54	924
A199	B183	C225	D327	E193	F175	G162	H418	I151	J139	2172

A16 To find a part of a number, given the ratio of the part to the number and the number. (The fraction may be in terms of fractions or decimals.)

A69	B83	C18	D125	E45	F28	G64	H78	I114	J52	676
A283	B439	C145	D278	E152	F110	G196	H513	I313	J215	2644

A17 To find a number, given a part of it and the ratio of that part to the whole.

A24	B69	C10	D42	E34	F13	G9	H5	I27	J49	282
A44	B170	C38	D45	E49	F33	G26	H7	I54	J103	569

A18 To divide a quantity into parts having a given ratio, given the quantity and the ratio.

A12	B7	C5	D2				H28	I13	J10	77
A16	B7	C7	D3			G1	H35	I14	J14	97

A19 To find a member of a ratio, given two members of one ratio and one member of another ratio equal to the first. (Inverse ratio included.)

A49	B181	C76	D116	E80	F36	G115	H98	I121	J110	982
A114	B211	C124	D145	E95	F74	G131	H153	I165	J152	1364

A20 To find the ratio of items to total, given a series of items.

	B1						H1			2
A1	B1		E1		G1		H7		J1	12

A21 To compare pairs of quantities by ratios, given the pairs of quantities.

		C5		E14		G3	H40	I5	J6	73
A1		C20	D4	E15	F1	G11	H46	I7	J12	117

A22 To find the largest quantity which will be contained equally in two or more given quantities.

	B7									7
	B7									7

A23 To find the least quantity which will contain exactly each of two or more quantities.

	B9									9
	B9									9

A24 To draw to scale, or to represent graphically in tables.

A8	B14	C39	D42	E23	F31		H47	I31	J8	243
A12	B22	C68	D60	E27	F33		H48	I38	J9	317

A25 To interpret tables, graphs, or diagrams, given completed graphs, tables, or diagrams.

A26	B19	C61	D23	E30	F26	G12	H82	I7	J7	293
A30	B21	C79	D26	E31	F28	G20	H91	I9	J14	349

B. ACTIVITY PROBLEMS

B1 Buying and selling,¹ simple cases.

a. To find the total price:²

1. given the number of units and price³ per unit.

A67	B92	C51	D206	E90	F29	G107	H179	I212	J162	1195
A391	B473	C348	D625	E433	F289	G472	H424	I440	J577	4472

2. given the number of units and the price per unit of another denomination.

A20			D3	E8		G15	H7	I9	J10	72
A25	B3	C1	D7	E8		G27	H15	I17	J27	130

b. To find the number of units:

1. given the total price and price per unit.

A9	B60	C42	D71	E15	F11	G10	H33	I33	J36	320
A14	B83	C65	D82	E22	F12	G15	H55	I38	J56	442

¹Descriptions of quantitative relations given below are expressed in terms of buying. In some cases changes in terminology would be necessary if the activity were to be considered from the standpoint of selling.

²"Total price" is used to designate the amount received for several units of the same commodity rather than the amount received for several commodities.

³"Price" is used to designate the quantity taken as a basis of computation. Usually "price" refers to the value or worth of a unit rather than a specified number of units. "Price" is often limited by the qualifying terms cost, selling, marked, and list.

2. given the total price and the price per unit in another denomination.
- | | |
|----|---|
| J1 | 1 |
| J1 | 1 |
3. received in exchange of commodities, given an amount of each commodity and the unit for each.
- | | |
|----|---|
| C2 | 2 |
| C2 | 2 |
4. given the price per unit of each of two commodities, the total price of both, and the ratio of the number of units of one to the number of units of the other.
- | | |
|----|---|
| E1 | 1 |
| E1 | 1 |
5. given the margin,⁴ per unit and the total margin.
- | | | | | | | | |
|----|----|----|----|----|----|----|----|
| E1 | 1 | | | | | | |
| A1 | B3 | C5 | D2 | E2 | G1 | I1 | 15 |
- c. To find the price per unit:
1. given the total price and the number of units.
- | | | | | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|------|-----|-----|-----|
| A19 | B22 | C10 | D19 | E20 | F7 | G12 | H88 | I9 | J15 | 221 |
| A63 | B93 | C59 | D48 | E62 | F19 | G42 | H163 | I27 | J67 | 643 |
2. given the total price and the number of units in another denomination.
- | | | | | |
|----|----|----|----|---|
| A1 | G1 | 2 | | |
| A3 | F1 | G2 | J1 | 7 |
3. in exchange of commodities, given the number of units of each commodity and the price per unit of one.
- | | |
|----|---|
| B2 | 2 |
| B2 | 2 |
4. given the number of units of each, the combined price of both, and the ratio of the price of the one to that of the other.
- | | |
|----|---|
| E1 | 1 |
| E1 | 1 |
- d. To find the amount to be received for several items, given the price of each.
- | | | | | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| A7 | B3 | C3 | D4 | E19 | F8 | G3 | H13 | I1 | J3 | 64 |
| A22 | B22 | C15 | D37 | E72 | F43 | G28 | H28 | I12 | J51 | 330 |
- e. To make change, given an amount of money and the price of a commodity.
- | | | | | | | | | | |
|-----|-----|-----|-----|-----|----|----|-----|-----|-----|
| A26 | B23 | C68 | D15 | E13 | I1 | J1 | 147 | | |
| A28 | B36 | C96 | D33 | E36 | G3 | H3 | I11 | J12 | 258 |
- f. To find the margin or loss given the cost price and the selling price.
- | | | | | | | | | | | |
|-----|------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| A1 | B1 | F1 | H3 | J1 | 7 | | | | | |
| A64 | B102 | C75 | D89 | E13 | F27 | G46 | H32 | I33 | J77 | 558 |
- g. To find the total margin or total loss:
1. given the number of units and the margin or loss per unit.
- | | | | | | | | | | | |
|-----|----|----|----|-----|----|----|----|----|----|----|
| A6 | D1 | H3 | J1 | 11 | | | | | | |
| A15 | B6 | C4 | D1 | E12 | F2 | G8 | H3 | I4 | J8 | 63 |
2. given the unit cost, the unit selling price, and number of units.
- | | | | | | | | | | | |
|----|-----|----|----|----|----|----|----|----|----|----|
| A1 | B10 | C1 | D2 | E3 | F1 | G5 | I1 | J3 | 27 | |
| A3 | B14 | C1 | D2 | E6 | F1 | G6 | H2 | I3 | J9 | 47 |
- h. To find the margin or loss per unit, given the total margin or loss and the number of units.
- | | | | | | | | | |
|----|----|----|-----|----|----|----|-----|----|
| D1 | 1 | | | | | | | |
| A2 | B1 | C3 | D30 | E2 | F3 | G1 | H13 | 55 |

⁴Margin is a term used to represent the difference between the cost price and the selling price and therefore is a substitute for the words "gain" and "profit" as they are commonly used.

B2 Buying and selling, more complex types.

a. To find the selling price:

1. given the rate⁵ of discount or loss, and the price.

A70	B18	C5	D83	E13	F52	G119	H106	I41	J14	521
A82	B25	C14	D86	E31	F64	G128	H140	I62	J35	667

2. given the rate of advance or margin and the price.

A11	B19	C1	D10		F6	G8	H25	I8	J5	93
A28	B44	C11	D13	E8	F8	G17	H39	I12	J14	194

3. given the rate of two or more successive discounts and the price.

A6	B12	C2	D25	E5	F47	G37	H15	I1	J12	162
A9	B13	C7	D36	E10	F49	G38	H16	I11	J17	206

4. given the price, rate of advance or margin, and rate of discount or loss.

A1	B2		D2			G1			J3	9
A2	B2		D2	E1		G2			J3	12

5. given the rate of commission, discount, margin, or loss and the amount of commission, discount, margin, or loss.

	B1					G1		I1	J2	5
	B1					G1		I1	J3	6

6. given the price and the amount of commission or discount.

				E2						2
A6	B18	C9	D14	F4	F4	G9	H5	I1	J11	81

b. To find the amount of margin, loss, commission or discount:

1. given the total price and the rate of margin, loss, commission or discount.

A49	B26	C5	D104	E6	F16	G29	H34	I9	J10	288
A91	B76	C30	D140	E28	F30	G53	H70	I25	J50	593

2. given two or more successive discounts and the total price.

	D1									1
	D4					G1				5

3. given the total price and the selling price.

A1										1
A3										3

c. To find the rate of margin, loss, discount, advance or commission:

1. given the total price and the amount of margin, loss, discount, advance or commission.

A65	B11	C8	D2	E14	F17	G3	H1	I2	J5	128
A122	B78	C63	D93	E30	F36	G30	H14	I23	J60	549

2. given the cost price in terms of two successive rates of discounts and the list price, and the selling price in terms of a single rate of discount and the list price.

								I16	J2	18
A2								I21	J3	26

3. given the total price and the selling price.

A24		C11	D61	E10	F29	G62	H24		J26	247
A32	B2	C13	D61	E11	F29	G65	H24		J29	266

d. To find the price:

1. given the selling price and the rate of discount or loss.

A1	B17	C2	D2	E5	F13	G4		I3	J6	53
A2	B30	C8	D2	E7	F14	G4		I5	J20	92

2. given the amount of margin, loss, commission, or discount and the rate of margin, loss, commission, or discount.

A8	B19	C7	D2			G1	H5		J10	52
A12	B34	C8	D2	E2		G3	H5	I2	J12	80

⁵Rate may be expressed in terms of percent or as a fraction.

3. given the selling price and rate of margin.

A2	B24		D2	E4	F29	G6		I9	J6	82
A10	B37	C5	D2	E5	F30	G7		I16	J17	129

4. given the selling price and two or more successive discounts.

		C1		F2		I2		5
	B1	C1	E1	F2		I2		7

e. To find the amount due the agent or agents, given the number of units, the price per unit, and the rate of commission. (Agent purchases commodity.)

A1	B10	C3		G2		I2		18
A1	B12	C4		G2		I2		21

f. To find the equivalent single discount in percent, given two or more successive rates of discount.

	C1	E1		2
	C1	E1		2

g. To find one of two or more successive discounts, given the list price, one or more of the successive discounts in percent, and the net price.

	J1	1
	J1	1

B3 Carrying on a business.

Note: Types listed under "carrying on a business" are similar in certain respects to those found under the activity of "buying and selling," but in general the following distinction prevails. The problems under B1 and B2 are those in which a purchaser is explicitly involved and in which he may be expected to be interested, at least to the extent of checking the solution by the seller. The problems under B3 are those which in general only the one carrying on the business will encounter. The degree of magnitude of the quantities of the problem and the terminology were also used as criteria. In cases where the distinction is not obvious, a footnote indicates similarities or differences.

a. To find the selling price:⁶

1. (a) given the cost price, rate of net profit, and rate of overhead. (Profit and overhead are figured on the cost price.)

	H1	1
	H1	1

(b) given the cost price, rate of net profit, and rate of overhead. (Profit and overhead are figured on selling price.)

	E1	1
	E1	1

2. (a) given the cost price, rate of net profit, and the overhead. (Profit is figured on cost.)

A4		D1		G1		6
A6	B4	D4		G2	J1	17

(b) given the cost price, rate of net profit, and the overhead. (Profit is figured on selling price.)

B1	C1	E1		3
----	----	----	--	---

b. To find the total receipts, given the total cost and the net profit.

	E1	1
--	----	---

c. To find the overhead, given the cost price or selling price and the percent of overhead.

		E6	G4	10	
	C1	E17	G9	I1	28

d. To find the rate of overhead, given the cost price or selling price and the overhead.

	G4	J1	5
--	----	----	---

⁶General terminology and "overhead" are the factors which distinguish this classification from B2a.

e. To find the net profit or loss:

1. given the cost price, overhead, and selling price.

				E1			H3			4
A5	B7	C1	D1	E20	F2	G6	H10	I6		58

2. given the cost price, rate of overhead, and selling price. (Overhead is figured on the cost.)

			D10							10
	B1	C2	D10	E3		G1				17

3. given the cost price, the rate of overhead, and the selling price. (Overhead is figured on the selling price.)

				E4						4
A2	B1	C1		E4	F3	G4				15

4. given the total costs and total receipts.

				E1		G1	H1			3
A8	B8	C9	D17	E25	F1	G21	H8	I4	J10	111

5. given the itemized costs and total receipts.

				E3			H1			8
A1	B3			E3	F2		H2	I2		49
A4	B9	C6	D12	E12						

6. given the rate of gross profit, the rate of overhead, and the selling price. (Overhead and gain are figured on the selling price.)

						G5				5
				E1		G5				6

7. given the rate of gross profit, the overhead, and the selling price. (Gross profit is figured on the selling price.)

						G2				2
						G3				3

8. given the gross profit and the overhead.

						G2		J1		3
--	--	--	--	--	--	----	--	----	--	---

f. To find what percent the net profit is of the cost price or selling price, given the cost price, overhead, and the selling price.

	A1			E2		G3				6
	A5			E4		G4		J1		14

g. To find what percent the net profit is of the cost price or selling price, given the net profit, and the total receipts, original outlay, or amount invested.

				E11	F38	G2				51
	B4	C1	D8	E31	F41	G6		J1		92

h. To find what percent the profit or loss is of the cost price or selling price, given the profit or loss and the cost price or selling price.⁷

							H2	I1		4
A3	B7	C4	D1	E9	F1	G7	H6	I1		39

i. To find the gross profit:

1. given the cost price and total receipts.

				E10		G1				11
--	--	--	--	-----	--	----	--	--	--	----

2. given the cost price or selling price and rate of gross profit.

						G4		J1		5
--	--	--	--	--	--	----	--	----	--	---

j. To find the rate of gross profit:

1. given the wholesale price (cost price) and the retail price (selling price).

				E1				J1		2
				E2				J1		3

2. given the total receipts, total amount invested or total costs, and the amount of gross profit or gross income.

						G2				9
	B1			E6		G2				

⁷This classification differs from B2c1 in size of quantitative terms and in terminology.

k. To find the amount invested:

1. given the itemized costs.

	E1								
	D1	E3						J1	5

2. given the rate of profit on the investment and the net profit.

B1			F1			I1			3
B1		E1	F2	G1		I2			7

l. To find the profit on an investment, given the amount of the investment, the rate of profit per unit of time and the time.

B1			F1			I38			40
B2		E1	F2	G1		I41			47

m. To find the profit per person on the basis of investment, given the amount invested by each person and the profit on the total investment.

B1	C1								2
B2	C1								3

n. To find the commission:

1. given a series of commodities, the number of units in each series, the unit price of each commodity, and the rate of commission.

	C1	E1	G1						3
A5	C1	E1	G1	H1					9

2. given the total sales, the expenses, and the rate of commission charged.

B1									1
B1						H1			2

3. given the total cost, the expenses, and the amount remitted to the agent

	D1								1
	D1								1

4. given the proceeds remitted by the agent, the rate of commission, and expenses.

A1									1
A1									1

o. To find the value of goods to be sold, given the rate of commission and the amount of commission to be earned.

A1	B3		E1			J2			7
A1	B4		E2			J5			12

p. To find the amount remitted to the agent given the selling price, the rate of commission and expenses.

					G1				1
A1	C1	D1	E1		G3		J1		8

q. To find the value of goods sold (selling price), given the net proceeds, the rate of commission and the expenses.

B2				F1			J3		6
B2				F1			J3		6

r. To find the net proceeds. (Wholesaler's point of view.)

1. given the amount of the sales and the rate of commission.

B2	C1	D30	E1	F7		I1			42
B3	C3	D32	E3	F10		I2			53

2. given the amount of sales, rate of commission and expenses.

A1	B4		E2		G1		J2		10
A2	B16	C7	D3	E4	G6	H1	J3		42

3. given the commission, rate of commission and expenses.

A1	B1								2
A1	B1								2

s. To find the rate of commission:

1. given the cost price or selling price, the expenses, and the amount remitted to the agent.

A1			D1						2
----	--	--	----	--	--	--	--	--	---

2. given the proceeds and the amount of sales.

	E1		1
	E2	F1	3

3. given the amount remitted to the owner or dealer, the amount of sales, and the expenses.

A1			1
A1		D1	2

B4 Borrowing, lending, and saving money.

a. To find the interest or discount, given the amount loaned, the rate of interest or discount, and the time or term.

A183	B68	C57	D343	E70	F394	G234	H168	I346	J251	2114
A195	B98	C79	D361	E86	F426	G245	H232	I374	J270	2366

b. To find the total interest received, given the rate of interest, the amount loaned, the term for compounding the interest, and the total time.

	B2	C1	D1	E9		J4	17
A13	B2	C4	D1	E9		J4	33

c. To find the exact interest, given the date the loan was made, the date the loan was due, the loan, and the rate of interest.

A11	B5	C2	D2	E8	F31		J6	65
A11	B5	C3	D2	E8	F31		J6	66

d. To find the amount loaned:

1. given the interest or discount, the rate, and the time.

A1	B2	C2	D5	E10	F10		H5	I5	J3	43
A1	B4	C2	D5	E10	F12	G3	H7	I5	J3	52

2. given the amount due, the time, and the rate of interest.

	B2			E5	F2		J1	10
	B2			E5	F2		J1	10

e. To find the face of a note, given the rate of discount, the proceeds, and the term of discount.

	E2		J5	7
	E2		J6	8

f. To find the amount due:

1. given the amount loaned, the rate of interest, and the time.

A2	B48	C17	D120	E66	F6	G18	H6	I10	J13	306
A9	B70	C43	D122	E71	F9	G26	H19	I39	J29	437

2. given the amount loaned, the rate of discount, and the time.

				E7	F8		J2	17
		C1		E7	F9	H6	J2	25

3. given the rate of interest, the amount loaned, the term for compounding the interest, and the total time.

A5			D3	E19	F22	G5	H10	I21	J1	86
A5	B2	C3	D3	E20	F25	G6	H10	I21	J1	96

4. at a given time, given one or more deposits, the date of each deposit, the rate of interest, and the term for compounding the interest.

A10	B1	C11	D20	E3	F1	G15		I2		63
A10	B3	C12	D21	E3	F2	G15		I3		69

g. To find the balance due, given the amount loaned, the time of interest payments, the partial payments, the total time, and the rate of interest.

A6		C2	D5			G1	H1	16	J13	34
A6	B3	C2	D5			G1	H1	16	J14	38

h. To find the proceeds, given the face of the note, draft, or trade acceptance, term of discount, rate of discount and time.

A11	B4	C1	D21	E20	F15	G28		I63	J15	178
A17	B20	C15	D34	E22	F17	G29		I72	J20	246

- i. To find the balance due at a given time, given a series of deposits, the time of each deposit, withdrawals, rate of interest, and the term for compounding the interest.

A3	C3	D1								7
A3	C3	D1								7

- j. To find the rate of interest or discount, given the amount loaned, the amount of interest or discount, and the time.

C1	D9	E14	F61		H8	16	J2	101
C1	D10	E15	F63		H14	18	J3	114

- k. To find the time, given the amount loaned, the rate of interest, and the amount of interest. (Reductions of time elements.)

A1		D9	E9	F1			11	21
A1	B3	D9	E11	F1			11	26

B5 Keeping accounts.

- a. To find the total of a bill or invoice, given an item or series of items, the number of each, the price of each, and the terms.

A4	B13	C7	D15	E41	F7	G3	H9	I24	J2	125
A17	B16	C15	D30	E46	F31	G3	H9	I25	J5	197

- b. To find the total value, given an inventory, and value of each item.

D1	E1		H2	4
D2	E1		H2	5

- c. To make out a bill or invoice, given an item or a series of items, the number of each, the price of each, the names of the purchaser and seller, and the terms.

A2	B7	C4	D5	E1	F14	G20	H1	I45	J2	101
A14	B35	C15	D51	E8	F38	G38	H2	I63	J14	278

- d. To make out a bank deposit slip, given two or more checks, an amount of bills, and several coins.

A4		C2		E4	F6	G3		I8	27
A4		C2	D1	E4	F6	G3		I11	31

- e. To make a monthly statement, given the items bought, the credits allowed, the purchaser, and the seller.

C3							I1	4
C3	D3						I1	7

- f. To make a contract, given the agreement, the consideration, the parties concerned, and the witnesses.

C2			E1	3
C2			E1	3

- g. To make out an inventory, given a series of items, the number in each series, and the value.

H1	I1	2
H1	I1	2

- h. To receipt a bill when paid, given the bill and the payment.

A8	B27	C1	D39	E7	F1	G14	H1	I12	J1	1
									J13	123

- i. To write a receipt, given the amount for which payment was received and the name of the payer.

A4	B3	C12	D6	E5			H1	17	38
A4	B5	C15	D6	E5			H1	18	44

- j. To write a note or trade acceptance, given the amount, rate of interest, payee, payer, and time.

A7	B4	C3	D2	E2	F8	G1	H4	I1		32
A10	B6	C8	D4	E2	F14	G3	H4	I15	J5	71

- k. To write a check, given the name of the bank, the amount of the check and the payee.

A5	B1	C5	D4			G2				17
A7	B1	C17	D17	E1		G4	H4	I1		52

l. To write a draft, given the amount of the draft, the name of the person in favor of whom the draft is drawn, of the bank on which the draft is drawn, and the bank drawing the draft.										
A2	C3	D4	E1		G3	I3	J3			19
A2	C3	D4	E2		G4		J4			26
m. To keep a stub of a check book:										
1. given an original deposit and a series of checks.										
					G1					1
					G1	H1				2
2. given an original deposit, a series of checks, another deposit, and another series of checks.										
				E3	G6					9
				E3	G6					9
n. To keep a cash book, given receipts and expenditures.										
	B1	C2			G1	H1		J4		9
A5	B2	C8	D4	E2	F6	G8	H2	I22	J4	63
o. To keep an account, given purchases and payments, simple accounts.										
	C1				F6	G3				10
p. To indorse a check, given a check. (Drafts and notes included.)										
A2	C1	D2		F2	G1					8
A6	C1	D6	E1	F8	G4		I9	J4		39
q. To find the balance of a cash book, given expenditures and receipts.										
A2	B1	C6	D15	E22		G7	H2			55
A8	B2	C12	D19	E26	F11	G15	H3	I22	J2	120
r. To balance an account, given purchases and payments.										
1. simple accounts.										
				E2	F1	G6				9
	C1	D3		E3	F7	G9				23
2. complex accounts.										
						G5				5
						G5				5
s. To balance a bank account, given an original balance, a series of deposits, and a series of withdrawals.										
A2	C1	D1		F5	G7	H7	I10	J1		34
A2	C1	D3		F5	G7	H7	I14	J1		40
t. To find a balance, given the exchange of commodities.										
								J1		1
								J1		1

B6 Construction.

Note: Problems involved in the following activities were included in this classification: woodworking, sewing, cooking, building construction, and fencing. Costs of construction materials were included.

a. To find how many times a given pattern, border, design, or length is contained in a given length.										
A7	B15	C7	D7	E16		G38	H13	I9	J3	115
A26	B27	C15	D45	E18	F4	G51	H20	I18	J3	227
b. To find the amount of fencing, given the number of wires to be used in a dimension of the area.										
						G1				1
	B1					G1				2
c. To find the total number of units:										
1. given the dimensions of the unit, and the dimensions of the whole.										
A24	B17	C12	D21	E22	F9	G48	H8	I8	J27	196
A37	B28	C20	D44	E37	F30	G84	H13	I17	J59	369

2. given the number of wholes and the dimensions of each whole.

				G1	H1			2
	B2			G8	H1		J5	16

3. given the dimensions of the whole and the size of the unit.

A1				G9			J1	11
A1	B3		D1	E6	G14		I1	J6

4. given the dimensions of the whole, the size of the unit, and the allowance. (Allowance for openings, waste, matching, etc.)

						I3		3
	B3		D1	E1	G3	I10	J6	24

5. given the dimensions of the whole, the dimensions of the unit, and allowance for waste, etc.

				F3			I1	4
	C1	D3	E1	F3			I5	J1

d. To find the number of shingles needed:

1. given the number of shingles used per square or a given area, and the dimensions.

		E1	F2			I2		5
	D1	E1	F2			I2	J1	7

2. given the number of shingles used per square and the area to be covered.

		E1						1
		E1					J1	2

e. To find the total number of board feet, given a mill bill.

A2			D42			H25	I8	77
A5	B3	C1	D49	E4	F7	H36	I13	J23

f. To find the amount of paint needed to cover an area, given the area covered by a unit measure of paint and the total area to be covered.

	B1					G1		2
	B2		E4			G1		7

g. To find the number of rolls of paper needed, given the dimensions of the room, and the allowance for openings.

				F5				5
		D6		F5				11

h. To find the rim speed, given the number of revolutions per minute, and the diameter.

		H14	I2	16
		H14	I2	16

i. To find the number of revolutions per minute, given the rim speed, and the diameter.

				G1	H3			4
	A3			G1	H3	I1	J1	9

j. To find the total cost of construction, given the cost per unit and the number of units.

		B1					H1	I6	8
	A23	B47	C4	D50	E27	F19	G33	H4	I24

k. To find the cost per unit of construction, given the total cost and the number of units.

	A1	B1		E1	G1		I3	7
	A3	B1		D1	E1	F3	G1	H1

l. To find the number of units, given the total cost and the cost per unit.

		E4						4
		E4						4

m. To find the number of units, given the size of the whole, and the size of the unit.

	B1					H1		2
	B1		E9			H1		11

B7 Travel, transportation, and communication.

Note: This type of problem includes travel by any means such as automobile, train, etc. It also includes transportation by truck, train, express, parcel post, or by any other means. Communication of any type may be included here, such as mail, telephone, telegraph, or radio.

a. Travel.

1. To find the distance:

(a) given the time and the rate.

	B11	C2	D8	E13	F5	G4	H19	I6	J1	69
A2	B23	C4	D8	E17	F9	G4	H29	I9	J6	111

(b) between two places, given the rate of travel, the time taken to travel the distance, the number of stops and the time for each stop.

	D1	1
	D1	1

2. To find the distance traveled per unit of time:

(a) given the total distance and the total time.

A2	B3	C5	D13	E7	F14	G16	H20	I2	J2	84
A3	B7	C11	D17	E10	F30	G22	H25	I8	J3	136

(b) given the distance between two places, the time taken to travel the distance, and the time spent for stops.

	G1	H1		2
	G1	H1	I1	3

3. To find the time:

(a) given the distance and the rate.

	B4	C4	D2	E10	F5	G5	H5	I8	J1	44
	B7	C4	D2	E17	F7	G5	H8	I8	J2	60

(b) between two stations, given one station in one time belt and another station in another time belt. (Eastward travel.)

	B1	1
	B1	1

(c) between two stations, given one station in one time belt and another station in another time belt. (Westward travel.)

	B1	1
	B1	1

b. Transportation.

1. To find the amount hauled by the same power over a good road, given the power, the amount hauled on a poorer road, and the ratio of the amount hauled on the poorer road to that which can be hauled on a better road.

	E1	1
	E1	1

2. To find the number of trips needed to haul a given amount over a good road with the same power used on a poorer road, given the power, the amount to be hauled, the amount hauled per load on the poorer road and the ratio of that load to the load hauled on the better road.

	E2	2
D1	E3	4

3. To find the cost:

(a) of sending a commodity or commodities by parcel post, given the rate of the article for a given zone, and the weight.

A26	B7	C27	D22	E1	F22	G1		J3	109
A27	B7	C28	D22	E1	F22	G1	I3	J16	127

(b) of shipping commodities by express, given the rate, the weight, and distance.

A6	B2	C2	D2	E11				J1	24	
A9	B4	C7	D5	E12	F1		H1	I1	J2	42

(c) of shipping small commodities, given the cost per pound, weight, or size.

A4	4
A4	4

(d) of hauling bulk commodities, given the total number of units and the cost per unit.

B1	C1	D1	E1	I1	1
				I5	9

(e) of carrying a load of equal weight over a good road, given the cost of power per mile on a poorer road, the distance traveled, and the ratio of the load the same power can haul on the good road.

E1	1
E1	1

(f) To find the cost per unit of hauling or transportation, given the total cost and the number of units.

A3	C3	D1	I2	3
			I5	12

4. To find the total cost of an article sent by parcel post, given the weight, the rate for the zone, the value of the article or articles, and the rate of insurance.

C2	E1	3
C15	E1	16

5. To find the freight rate, given the amount of freight charges, and the weight.

C1		1
C1	D1	2

c. Communication.

1. (a) To find the amount charged for collection of a draft, given the face value, and the rate charged.

D3	E2	J5	10
D3	E5	J7	15

(b) To find the rate of exchange, premium, or discount, given the face value of a money order or draft, and the total cost.

I2	2
I2	2

(c) To find the proceeds, given the amount of the draft, money order, or bill, and the rate charged for collection.

E2	I1
E8	I1

2. To find the cost:

(a) of a money order or draft, given the amount sent, and the rate charged.

C1	D10	E12	F6		H2	I45	J16	92	
A17	C1	D10	E19	F6	G1	H2	I45	J26	127

(b) of mailing letters, newspapers, etc., given the rate of postage per unit and the number of units. (Unit may mean letters or weight.)

A14	D2	F2	18
A20	D11	F4	35

(c) To find the cost of sending a telegram or telephoning, given the number of units, a rate for a given number of units, and an added rate for each additional unit.

B1	D3	4
B1	D3	4

B8 Municipal and federal activities. (Excluding taxation.)

a. Municipal activities.

1. To find the per capita expense of a community activity, given the total cost and the population. (Total number of persons.)

	E4	F12	G1	H2	I1	20
C1	E6	F12	G1	H2	I1	23

2. To find the number of lives saved, given the death rate, the decrease in percent (due to an applied remedy) and the population.

E1	1
E1	1
3. To find the number of lives saved, given the death rate at one period, the death rate at a later period, and the population at each period.

E1	1
----	---
4. To find the per unit cost of a community activity, given the total cost, and the number of units.

A2			G7	I1	10
A2	D1	F4	G7	I1	15
5. To find the per capita loss, given the valuation of property destroyed, and the total population.

E1	1
----	---
6. To find the total cost of a community activity, given the number of units or the total population, and the cost per unit or per person.

E1	G2	3
E2	G2	4
7. To find the death rate per a given number, given the number of deaths, and the total number of persons.

C1	E3	F1	5
C17	E3	F1	21

b. Federal activities.

1. To find the number of years of peace needed to pay for a year of war, given the amount saved during a year of peace, and the total amount spent during a year of war.

H1	1
----	---

B9 Insurance.

a. To find the premium:

1. given the face value of the policy, the rate of insurance, and the term.

A4	B5		D22	F4	G2	H9	I36	J1	83	
A19	B10	C1	D29	E1	F13	G2	H12	I37	J9	133
2. given the valuation of the property, the ratio of that value which was accepted for insurance, rate and term.

A1	B1	C3	D1	E5	F1	G1	H2	I4	J3	22
A5	B1	C5	D2	E6	F1	G5	H2	I5	J5	37
3. given the face value of the policy, the original rate of insurance, the percent of decrease due to the installation of protection devices.

E1	1
----	---

b. To find the total premium, given itemized values and respective rates, and an added rate for an additional risk.

E1	1
E1	1

c. To find the amount of insurance, given the rate and the premium.

B3	C1		G1	I14	J8	27
B3	C2	E1	G1	I16	J12	35

d. To find the rate of insurance, given the premium, the face value of the policy, and the term.

A1		D3		I12	J3	19	
A2	B1	D6		H2	I14	J4	29

B10 Personal investments such as life insurance, real estate, stocks and bonds. (Stocks include investments in building and loan associations.)

a. Life insurance.

1. To find the premium on a life insurance policy, given the table of annual premiums based on \$1000.00, the kind of policy and time.

A18			D1	E3	F4	G6		I9	J1	42
A21	B2	C6	D1	E3	F5	G6		I11	J1	56

2. To find the difference in the amount paid in and the amount received, given the age at which the policy was taken, age at maturity, kind of policy, and table of premiums.

B5	E1	F4	G3	13
B7	E1	F4	G3	15

3. To find the cost of protection, given the face value of the policy, the premium per year, the number of years, and cash surrender value.

G2	2
G2	2

b. Real estate.

1. To find the profit, given the original cost, the selling price, other necessary costs, receipts, and the time.

D2	F2	H2	6	
D6	E1	F2	H2	11

2. To find the loss, given the original cost, the selling price, other necessary costs, losses, receipts, and the time.

A1	E1	2
----	----	---

3. To find the rate of profit or loss, given the cost price, the selling price, and expenses. (Selling price includes receipts. Expense includes added cost or losses.)

B1	D1	E2	F1	5
B2	D6	E3	F2	13

4. To find the cost per front foot, given the total cost and feet of frontage on street.

E1	1
E4	4

5. To find the rate of profit, given the cost, the rent, and the expenses and losses.

B3	D5	E2	F1	G1	I1	13		
A3	B3	C1	D6	E2	F3	G1	I2	21

6. To find the amount of rent necessary to make a given rate on an investment, given the amount of the investment, and the expenses.

A1	B2	C1	D1	E1	G2	11	
A1	B2	C1	D4	E3	F1	G2	14

7. To find the total price, given the rent, expenses, and rate of profit on the investment.

E1	1
E2	2

8. To find the net income on an investment, given the amount invested, the profit, and the expenses.

D1	F1	2
----	----	---

c. Stocks and bonds.

1. To find the dividend, given the amount of the bonds, or stock, the interest period, and the rate of interest.

A10	B1	C3	D15	E14	G18	H10	I7	J8	86	
A16	B2	C4	D20	E16	F1	G28	H55	I9	J18	169

2. To find the dividend, given the total cost of bonds, rate of dividend, and the quotation.

G1	J2	3
G1	J2	3

3. To find the amount of dividend, given the number of shares or bonds, the par value per share or bond, the rate of dividend, and time.

A3	D14	E5	I2	24		
A4	C1	D14	E5	H7	I3	34

4. To find the cost of bonds or stock, given the quotation, par value per share, brokerage, and number of shares.

A1	C3	D23	E14	G6	H15	I1	J14	76
	C3	D23	E15	G6	H15	I1	J17	81

5. To find the cost of a bond or bonds, given the amount of the bond or bonds, the interest, the time, quotation, and brokerage.
- | | | | | | | | | |
|----|--|--|--|--|-----|--|----|----|
| A2 | | | | | G1 | | J3 | 6 |
| A2 | | | | | G11 | | J3 | 16 |
6. To find the cost of bonds, given the dividend, the rate of dividend, brokerage, and rate of premium.
- | | | | | | | | | |
|--|--|--|--|--|----|--|--|---|
| | | | | | G1 | | | 1 |
| | | | | | G1 | | | 1 |
7. To find the total cost or amount of stock, given the number of shares and quotation or par value per share.
- | | | | | | | | | | |
|----|----|----|----|----|--|-----|-----|-----|----|
| A6 | B1 | C2 | D2 | | | H3 | I9 | J10 | 33 |
| A8 | B1 | C3 | D9 | E2 | | H12 | I10 | J30 | 75 |
8. (a) To find the profit or loss, given the number of shares of stock, the brokerage, the quotation at which it was bought, and the quotation at which it was sold.
- | | | | | | | | | | |
|----|--|----|----|----|----|----|----|----|----|
| A4 | | C3 | D6 | E2 | G2 | | I6 | J1 | 24 |
| A4 | | C3 | D6 | E2 | G2 | H2 | I6 | J1 | 26 |
- (b) To find the profit or loss, given the number of shares of stock, the brokerage, the quotation at which it was bought, the quotation at which it was sold, and the rate of dividend received.
- | | | | | | | | | | |
|----|--|----|----|----|----|--|--|--|---|
| A1 | | C1 | D1 | E1 | G1 | | | | 5 |
| A1 | | C1 | D1 | E2 | G1 | | | | 6 |
9. To find the profit, given a cost of stock or bonds, the amount of dividend received, and the selling price.
- | | | | | | | | | | |
|--|--|--|--|--|--|----|--|--|---|
| | | | | | | H1 | | | 1 |
| | | | | | | H1 | | | 1 |
10. To find the number of shares or bonds, given the amount of dividend, the rate of interest, and the par value per share or bond.
- | | | | | | | | | |
|----|--|--|--|--|----|--|----|----|
| A1 | | | | | G2 | | J4 | 7 |
| A1 | | | | | G2 | | J9 | 12 |
11. To find the number of shares or bonds, given the total cost of stocks or bonds, the quotation, and brokerage.
- | | | | | | | | | |
|----|--|----|--|--|----|--|----|----|
| | | C3 | | | | | J7 | 10 |
| A1 | | C3 | | | G2 | | J8 | 14 |
12. To find the number of shares or bonds, given the total cost or total amount of bonds, and the quotation, or par value per share or bond.
- | | | | | | | | | | |
|----|--|----|----|----|--|----|----|-----|----|
| | | C1 | D3 | | | | I3 | J5 | 12 |
| A2 | | C1 | D3 | E1 | | H4 | I3 | J19 | 33 |
13. To find the amount received for bonds or stocks, given the amount of bonds or stocks, the quotation and brokerage.
- | | | | | | | | | |
|--|--|--|--|--|----|--|-----|----|
| | | | | | G6 | | J5 | 11 |
| | | | | | G6 | | J11 | 17 |
14. To find the amount of bonds or stocks, given the percent of dividend, and the amount of the dividend.
- | | | | | | | | | | |
|--|----|--|--|----|----|--|----|----|----|
| | B1 | | | E1 | G3 | | I1 | J4 | 10 |
| | B1 | | | E1 | G3 | | I1 | J9 | 15 |
15. To find the amount of stocks or bonds, given the total cost, the quotation, and brokerage.
- | | | | | | | | | |
|--|--|--|--|--|--|--|----|---|
| | | | | | | | J2 | 2 |
| | | | | | | | J3 | 3 |
16. To find the amount received (amount remitted by agent after deducting his brokerage) given the number of shares or bonds, the quotation, and brokerage.
- | | | | | | | | | |
|----|--|----|----|----|--|--|----|----|
| | | C1 | D4 | E2 | | | I1 | 8 |
| A2 | | C1 | D4 | E2 | | | I1 | 10 |

17. To find the percent of profit, given the quotation, percent of dividend, brokerage and time.

B1	C6	F20	G4	I4	35
B1	C6	F20	G4	I4	35

18. To find the rate of profit, given the total cost of bonds or stocks, and the amount of profit.

A1				H2	J1	4
A2		E1	F1	H8	J3	15

19. To find the percent of profit or loss, given the amount of profit or loss, and the amount invested.

C1	E1				2
----	----	--	--	--	---

20. To find the rate of dividend, given the amount of dividend, and the amount of bonds or stocks.

B1		G1	H1	J1	4	
A2	B1	G1	H2	I2	J3	11

21. To find the amount of brokerage, given the total cost of bonds or stock, and the rate of brokerage.

H1		1
H1	J2	3

22. To find the cost or par value per share or bond, given the total cost and the number of shares or bonds.

C3		H2	J1	6
C3		H2	J2	7

23. To find the proceeds, given the quotation, rate of interest, and brokerage

A9	B2	D4		15
A9	B2	D4		15

B11 Personal activities involving wages and salaries.

a. Wages.

1. To find the amount of wages:

- (a) given the number of units, and the wage per unit.

A6	B11	C1	D6	E16	F3	G3	H32	I5	J14	97
A50	B37	C62	D27	E47	F22	G24	H95	I53	J38	455

- (b) given the price per unit for a given number of units, a higher price for added units, and a still higher price for more added units, and the total number of units.

B1	C1	E3	H1	1
			H7	12

- (c) To find the amount of wages earned in a given time, given an amount earned in a different length of time at the same rate.

E10	10
E10	10

2. To find the wages earned per unit, given the number of units and the total wage.

A1	B1	D3	E11	F1	G5	H4	I1	J6	33
A8	B7	D5	E18	F1	G11	H8	I4	J7	69

3. To find the number of units, given the total wages earned, and the wage per unit.

A1	B3	G3	H2	J2	11
A1	B3	G3	H2	J4	13

4. To find how much a group of men can earn in a given time at a given rate per hour, given a different number of men, and the total amount earned at the same rate per hour.

D2	2
D2	2

5. To find the wages earned by each person, given the total wages earned by the total number of persons, and the time each worked.

D4	12	6
D4	12	6

6. To find the amount of advance or decrease, given an original wage or payroll, and the percent of advance or decrease.

			D4							4
A1	B1		D4	E1						7

7. To find the wage or amount of payroll, given an original wage or an original payroll, and the percent of advance.

	B4		D7		G2	H6				19
	B5		D7	E1	G2	H6		J1		22

8. To find the smallest number of coins and bills necessary for a payroll, given a number of workmen, the wage per unit, and the number of units.

			E2							2
			E2							2

9. To find the rate of advance or rate of reduction, given an original wage, and the wage after the advance or reduction.

	A4		D8							12
	A4		D8							12

b. Salaries.

1. To find a salary, given an original salary and rate of increase.

		B1	D1							2
A1	B1		D4				I5			11

2. To find the total salary, given the number of units, and the salary per unit.

						H1				1
	C1	D2	E2	F1		H11	I1	J9		27

3. To find the salary per unit, given the total salary and the number of units.

						H1	I1			2
			E2			H1	I1			4

B12 Taxation, municipal, state, or national.

a. To find the amount of tax:

1. given the rate of taxation and assessed valuation, or quantity. (Note: Duties and poll tax included in this item.)

A14	B28	C1	D50	E8	F17	G31	H16	I56	J29	250
A39	B65	C43	D77	E22	F85	G37	H29	I66	J55	518

2. given the real value, the ratio of assessment to real value, and the rate of taxation.

		B2		D3		F1				6
A3	B3	C2	D9			F1				18

b. To find the rate, given the tax and assessment.

A16	B11	C1	D12			G9		I1	J8	58
A20	B15	C2	D18	E2	F3	G12		I1	J12	85

c. To find the total amount of assessment or quantity taxed, given the rate, and amount of tax or duty.

A2	B3		D1	E2		G1		I2	J9	20
A2	B4		D1	E2		G1		I2	J12	24

B13 Determining economy of two or more procedures. (This classification includes problems involving difference, saving, choice, and comparison.)

a. Difference.

1. To find the difference in unit costs, given different unit costs of two commodities. (This includes two qualities of the same commodity.)

				E5						5
A1	B3			E11		G1	H2			18

2. To find the difference in price per unit, given the number of units and the total price of one quality of a commodity and the number of units and the total price of another quality of the same commodity.

			D1	E2				J1		4
			D1	E3				J1		5

3. To find differences in amount, given two different unit costs for the same commodity or different commodities, and the total number of units.
- | | | | | | | | | | |
|----|----|----|-----|----|----|----|----|----|----|
| A2 | B2 | | E8 | F1 | G3 | H8 | I2 | J3 | 29 |
| A3 | B3 | D1 | E12 | F1 | G4 | H8 | I3 | J5 | 40 |
4. To find the difference in amount, given a total cash payment, and a given number of installments at a given payment each.
- | | |
|----|---|
| E1 | 1 |
|----|---|
5. To find the difference in the amounts of a bill, given different successive discounts, or different terms.
- | | |
|----|---|
| J2 | 2 |
| J2 | 2 |
6. To find the difference in number of units purchased for the same amount of money, given the amount of money and different prices for each of two qualities of the same commodity.
- | | |
|----|---|
| E1 | 1 |
| E1 | 1 |
7. To find the difference in units of time between two places, given two distances of different lengths, and the rate of travel.
- | | |
|----|---|
| E2 | 2 |
| E2 | 2 |
8. To find the difference in the rate of discount, given the marked price and the selling price of one commodity and the marked price and the selling price of another quality of the same commodity.
- | | |
|----|---|
| E1 | 1 |
| E1 | 1 |
9. To find the difference in the rate of travel, given the distance between two places, and the total time for each of two means of travel.
- | | | |
|----|----|---|
| E1 | 1 | |
| E1 | I1 | 2 |
10. To find the difference, given a selling price with a discount and a different selling price with a different discount.
- | | | | |
|----|----|----|---|
| A1 | E2 | 3 | |
| A1 | E2 | H1 | 4 |
11. To find the difference in amount of profit, given the amount received for a given number of units before spraying, and the amount received for a given number of units after spraying, and the cost of spraying.
- | | | |
|----|----|---|
| D4 | G1 | 5 |
|----|----|---|
12. To find the difference in amount of profit, given an amount of money invested in real estate with the cost, time, necessary expenses, rent per month, and selling price; and the same amount of money, drawing interest at a given rate for the same length of time.
- | | | | | | |
|----|----|----|----|----|---|
| | C1 | D2 | G1 | H2 | 6 |
| A1 | C1 | D3 | G1 | H2 | 8 |
13. To find the difference in amount of profit, given an amount of stock with rate of dividend, and the same amount invested in a bond and mortgage with rate of interest.
- | | |
|----|---|
| D1 | 1 |
|----|---|
14. To find the difference in interest due, given an amount of money, for a given time, at a rate of simple interest, and the same amount of money, for the same time, at the same rate but compounded.
- | | | | |
|----|----|----|---|
| D1 | E1 | 2 | |
| C1 | D1 | E1 | 3 |
15. To find the difference in amount of interest due, given an amount of money drawing interest for a given time at a given rate, compounded at a given period; and the same amount of money drawing interest for the same time at the same rate, but compounded at a given shorter period.
- | | | |
|----|----|---|
| E1 | H1 | 2 |
| E1 | H1 | 2 |

16. To find the difference in amount of interest, given the amount, time, rate of interest; and the same amount of money for the same time, but at a different rate of interest.
- | | | | | |
|----|----|----|----|---|
| C1 | E1 | H3 | I1 | 6 |
| C1 | E1 | H4 | I1 | 7 |
17. To find the difference in the cost of shipping crated and uncrated articles, given the rate of the crated article and the rate of the same article shipped uncrated.
- | | |
|----|---|
| E1 | 1 |
|----|---|
18. To find the difference in cost, given the total number of units traveled, the total cost by one method, and the cost per unit per person by another method, and the number of persons.
- | | |
|----|---|
| D1 | 1 |
| D1 | 1 |
19. To find the difference in cost of sending an amount of money, given the amount sent, the rate or charges by one method, and the rate or charges by another method.
- | | | | |
|----|----|----|---|
| D1 | E2 | 3 | |
| A2 | D1 | E2 | 5 |
20. To find the difference in premiums, given two buildings of equal value, different material, and different insurance rates.
- | | |
|----|---|
| E1 | 1 |
|----|---|
21. To find the difference, given an amount of a ten-year endowment policy, a twenty-year endowment policy and a table of annual premiums.
- | | |
|----|---|
| E1 | 1 |
| E1 | 1 |
22. To find the amount of difference, given a salary, plus a commission on sales over a certain amount; or a higher commission on all sales and no salary, and the total amount of sales.
- | | |
|----|---|
| D1 | 1 |
| D1 | 1 |
23. To find the difference in tax, given rate and assessed value in one area or at a given time and rate and assessed value in another area or at another time.
- | | | |
|----|----|---|
| E1 | H1 | 2 |
| E1 | H1 | 2 |
24. To find the difference in wages, given one wage and hour per day schedule, and another wage and hour per day schedule, and the time.
- | | |
|----|---|
| E3 | 3 |
|----|---|
- b. Saving.
1. To find the amount saved:
- (a) given the saving per unit and the number of units.
- | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|
| C1 | E2 | G1 | I2 | J1 | 7 | | | |
| A1 | B2 | C1 | E6 | G2 | H2 | I7 | J4 | 25 |
- (b) given the number of units saved, and the price per unit.
- | | | | | | |
|----|----|----|----|----|----|
| A3 | D1 | E1 | G1 | 1 | |
| | | | G4 | J1 | 10 |
- (c) given two different unit costs and the number of units.
- | | | | | | | | | |
|----|-----|----|----|----|----|----|----|----|
| B1 | D2 | F2 | G2 | H1 | J2 | 10 | | |
| B4 | C61 | D3 | E4 | F3 | G5 | H1 | J4 | 85 |
- (d) given the itemized list of original accounts and the itemized list of the same accounts reduced.
- | | |
|----|---|
| J1 | 1 |
| J2 | 2 |
- (e) given the cost per unit, a smaller cost per unit for a larger lot, and the number of units bought.
- | | | | |
|----|----|----|---|
| B1 | H1 | 1 | |
| | H1 | J1 | 3 |

- (f) given a price per unit, a smaller proportionate cost for a larger lot, and the number of units.

A6	B3	C1	D5	E1		G4	H8	I7	J3	38
A8	B3	C6	D6	E1	F1	G6	H15	I8	J3	57

- (g) given a cost price or selling price, or total quantity, and the percent or fractional part saved.

A2	B4			E2		G3	H1	I2	J2	8
						G6	H5	I7	J2	28

- (h) given an amount of insurance on a building, a rate; a lower rate due to installation of a safety device, and the term.

						F1				1
				E1		F1				2

- (i) given the amount of insurance, the term, a number of policies at a given rate for a long term, and a larger number of policies at a given rate for a shorter term.

				E2						2
				E2						2

- (j) given the cost price or selling price of one or more articles in each of two invoices, and the terms.

				E1		G4				5
				E2		G4				6

- (k) given an amount paid down, an amount paid in installments, the time and rate of interest; and the same amount down, with different amounts of installments for a different time, and no interest.

						H1				1
						H1				1

2. To find the percent saved, given the amount saved and the basic price.

A1	B3	C22			F1	G5			J1	33
----	----	-----	--	--	----	----	--	--	----	----

3. To find the amount saved if the commodity is home made:

- (a) given an itemized list of commodities and the cost per item without cost of labor; and given the total cost for the complete job including the cost of labor.

A6				E1		G2				9
A8	B1	C1		E2		G2	H1		J1	16

- (b) given an itemized list of commodities and the cost per item with the cost of labor allowed for; and given the total cost for the complete job.

A1				E1						2
A1	B1			E1		G1				4

4. To find the saving in premium, given the value of a building, the rate for a policy for one year, a cheaper rate for more than one year, and the time.

						G1			J1	2
A1					F1	G1			J1	4

5. To find the time saved, given the total time by one method of travel, and the total time by another method of travel.

	B1									1
	B1									1

c. Choice.

1. To find the most economical purchase, given the cost of a large unit, a larger proportionate cost of a smaller unit, and a still larger proportionate cost of a still smaller unit, and the number of units.

	B2		D1			G1	H1	I2		7
	B5		D1			G1	H1	I2	J1	11

2. To find the more economical procedure:

- (a) given the selling price of a commodity, and the choice of a single discount or two or more successive discounts.

A1	B2		D2			G1			J1	7
A1	B2		D2	E1		G2			J1	9

- (b) given the total number of units, the cost per unit of the whole commodity; or the cost per unit of one quality and the cost per unit of the other remaining quality of the commodity.
- | | |
|----|---|
| B1 | 1 |
| B1 | 1 |
- (c) given an amount of money borrowed for a given period of time at a given rate of interest; or a part of the amount borrowed paid in cash by the lender, and the remainder for the given time at a given rate of interest.
- | | |
|----|---|
| H1 | 1 |
|----|---|
- (d) given one cost and selling price and a different cost and selling price.
- | | |
|----|---|
| B1 | 1 |
| B1 | 1 |
3. To find the better selling price of the same commodity, given the selling price and discount in one case and a lower selling price in the second case.
- | | | |
|----|----|---|
| B2 | G1 | 3 |
| B2 | G1 | 3 |
4. To find the economical method of shipping, given the weight of an article and the rate charged for shipping in each of two or more modes of transportation.
- | | |
|----|---|
| C4 | 4 |
| C4 | 4 |
5. (a) To find the better investment, given the amount invested in each case, the interest in each case, and the time.
- | | | | |
|----|----|----|---|
| | G2 | H1 | 3 |
| D1 | G2 | H1 | 4 |
- (b) given one investment with amount of profit, and another investment with an amount of profit.
- | | |
|----|---|
| B1 | 1 |
|----|---|
- (c) given the amount invested, the rent, expenses, and time; and the same amount invested, for the same time at a given rate of interest.
- | | | | |
|----|----|----|---|
| B1 | C1 | D1 | 3 |
| B2 | C1 | D1 | 4 |
6. To find the better wage:
- (a) given one wage and hour per day schedule, and another wage under another hour per day schedule.
- | | |
|----|---|
| A1 | 1 |
| A1 | 1 |
- (b) given a wage per unit of time, the time, and a lower wage per unit of time for a longer time, and the time.
- | | |
|----|---|
| A1 | 1 |
| A1 | 1 |
7. (a) To find the better salary, given a salary per small unit of time, and another salary per larger unit of time.
- | | | |
|----|----|---|
| D1 | H1 | 2 |
|----|----|---|
- (b) given one salary at one time, another salary at a different time, and the ratio of the value of a dollar at the one time to the value at the other time.
- | | | |
|----|----|---|
| B1 | H2 | 3 |
| B6 | H2 | 8 |
8. To find the more economical purchase:
- (a) given a number of units of one quality (width, material, etc.) of a commodity and the price per unit; and a different number of units of a different quality of the same commodity, at a different price per unit.
- | | | | | |
|----|----|----|----|---|
| B1 | D1 | G2 | H1 | 5 |
| B1 | D1 | G2 | H1 | 5 |

(b) given different total prices, with different successive discounts. (May be the same total prices.)

C3	D2	5
C3	D2	5

9. To find the better offer, given an amount of insurance, the rate for a short term, the rate for a longer term, and the time.

A1		1
A1	B3	4

d. Comparison.

1. To compare the results of two or more procedures, given the procedures.

A2	B4	C2	D3	E1	F2	G10	H31	I11	J1	67
A5	B4	C2	D7	E4	F2	G11	H33	I15	J1	84

2. To compare areas, given the dimensions of each.

	E1	G2	H3	6
B1	E1	G2	H3	7

3. To compare costs of two or more commodities or items, given the cost of each per unit and the number of units.

C6	G1	H10	17	
C6	G1	H10	I1	18

4. To compare pairs of quantities by ratios, given the pairs of quantities. (Ratios may be in whole numbers, percents, or fractions.)

	H14	J1	15
C3	H14	J3	20

5. To find how many times as much it costs to send a given number of small money orders, at a given rate per small money order, than to send one large money order equivalent to the small ones, at a given rate per large money order.

H2	2
H2	2

APPENDIX B

The following questions concerning functional relationships are suggested for recognition as minimum essentials. The list is based upon the analysis of ten series of arithmetics but represents in part the judgment of the writer. In each case the question is: What calculations must be performed in order to find the quantity named, given the quantities specified?

- A1 To find totals by addition, given two or more items, values, etc.
- A2 To find the difference, given two items, values, etc.
- A3 To find the amount, or number needed, given a magnitude and the number of times it is to be taken.
- A4 To find the size of a part of a magnitude, given the magnitude and the number of parts into which it is to be divided.
- A5 To find how many times a stated quantity is contained in a given magnitude, given the quantity and the magnitude.
- A6a To find how many when reduction ascending is required, given a magnitude expressed in terms of a single denomination.
- A6b To find how many when reduction ascending is required, given a magnitude expressed in terms of two or more denominations.
- A7a To find how many when reduction descending is required, given a magnitude expressed in terms of a single denomination.
- A7b To find how many when reduction descending is required, given a magnitude expressed in terms of two or more denominations.
- A8a To find a dimension, given the area of a rectangle and one side.
- A10a To find the area, given dimensions of a square, rectangle, or parallelogram.
- A10b To find the area, given the base and altitude of a triangle.
- A10c To find the area, given the diameter of a circle.
- A11a To find the perimeter, given one side of any equilateral figure.
- A11b To find the perimeter, given two adjacent sides of a rectangle or parallelogram.
- A11c To find the circumference, given the diameter or radius of a circle.
- A12b1 To find the cubic contents, given the three dimensions of a rectangular solid, such as room, bin, woodpile, etc.
- A12b2 To find the cubic contents, given the area of one surface of a rectangular solid and the depth or altitude.
- A14 To find the average, given a series of items.
- A15 To find the ratio of one number to another, given the two numbers.
- A16 To find a part of a number, given the ratio of the part to the number, and the number.
- A18 To divide a quantity into parts having a given ratio, given the quantity and the ratio.
- A19 To find a member of a ratio, given two members of one ratio and one member of another ratio equal to the first. (Inverse ratio included.)
- A20 To find the ratio of items to total, given a series of items.
- B1a1 To find the total price, given the number of units and price per unit.
- B1a2 To find the total price, given the number of units and the price per unit of another denomination.

- B1b1 To find the number of units, given the total price and price per unit.
- B1c1 To find the price per unit, given the total price and the number of units.
- B1d To find the amount to be received for several items, given the price of each.
- B1e To make change, given an amount of money and the price of a commodity.
- B1f To find the margin or loss, given the cost price and the selling price.
- B1g2 To find the total margin or total loss, given the unit cost, the unit selling price, and number of units.
- B2a1 To find the selling price, given the rate of discount or loss, and the price.
- B2a2 To find the selling price, given the rate of advance or margin and the price.
- B2a3 To find the selling price, given the rate of two or more successive discounts and the price.
- B2a6 To find the selling price, given the price and the amount of commission or discount.
- B2b1 To find the amount of margin, loss, commission or discount, given the total price and the rate of margin, loss, commission or discount.
- B2c1 To find the rate of margin, loss, discount, advance or commission, given the total price and the amount of margin, loss, discount, advance or commission.
- B2c3 To find the rate of margin, loss, discount, advance, or commission, given the total price and the selling price.
- B2d3 To find the price, given the selling price and the rate of margin.
- B3e1 To find the net profit or loss, given the cost price, overhead, and selling price.
- B3e4 To find the net profit or loss, given the total costs and total receipts.
- B3e5 To find the net profit or loss, given the itemized costs and total receipts.
- B3g To find what percent the net profit is of the cost price or selling price, given the net profit, and the total receipts, original outlay, or amount invested.
- B3h To find what percent the profit or loss is of the cost price or selling price, given the profit or loss, and the cost price or selling price.
- B4a To find the interest or discount, given the amount loaned, the rate of interest or discount, and the time or term.
- B4f1 To find the amount due, given the amount loaned, the rate of interest, and the time.
- B4g To find the balance due, given the amount loaned, the time of interest payments, the partial payments, the total time, and the rate of interest.
- B5a To find the total of a bill or invoice, given an item or series of items, the number of each, the price of each, and the terms.
- B5q To find the balance of a cash book, given expenditures and receipts.
- B5s To balance a bank account, given an original balance, a series of deposits, and a series of withdrawals.
- B6a To find how many times a given pattern, border, design, or length is contained in a given length.
- B6c1 To find the total number of units, given the dimensions of the unit, and the dimensions of the whole.
- B6c3 To find the total number of units, given the dimensions of the whole and the size of the unit.
- B6j To find the total cost of construction, given the cost per unit and the number of units.
- B6k To find the cost per unit of construction, given the total cost and the number of units.

- B7a1(a) To find the distance, given the time and the rate.
- B7a2(a) To find the distance traveled per unit of time, given the total distance and the total time.
- B7a3(a) To find the time, given the distance and the rate.
- B7b3(a) To find the cost of sending a commodity or commodities by parcel post, given the rate of the article for a given zone, and the weight.
- B7c2(a) To find the cost of a money order or draft, given the amount sent, and the rate charged.
- B7c2(b) To find the cost of mailing letters, newspapers, etc., given the rate of postage per unit and the number of units. (Unit may mean letters or weight.)
- B10b1 To find the profit, given the original cost, the selling price, other necessary costs, receipts, and the time. (Real estate.)
- B10b5 To find the rate of profit on real estate, given the cost, the rent, and the expenses and losses.
- B10b6 To find the amount of rent necessary to make a given rate on an investment, given the amount of the investment, and the expenses.
- B10c1 To find the dividend, given the amount of the bonds, or stock, the interest period, and the rate of interest.
- B11a1(a) To find the amount of wages, given the number of units, and the wage per unit.
- B11a2 To find the wages earned per unit, given the number of units and the total wage.
- B11b2 To find the total salary, given the number of units, and the salary per unit.
- B13b1(c) To find the amount saved, given two different unit costs and the number of units.
- B13b1(f) To find the amount saved, given a price per unit, a smaller proportionate cost for a larger lot, and the number of units.
- B13b3(a) To find the amount saved if the commodity is home made, given an itemized list of commodities and the cost per item without cost of labor; and given the total cost for the complete job including the cost of labor.
- B13d1 To compare the results of two or more procedures, given the procedures.

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